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Calculations and Measurements on Crossflow Heat Exchangers

ROLAND GÖRTZ

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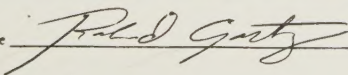


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Abstract The first part presents different mathematical models for calculating the effectiveness of thin walled plate crossflow heat exchangers operating with gases on both sides, either in single or in multipass arrangements. In all cases the calculations are made on the assumption that no mixing takes place within each pass. Between the different passes calculations have been performed both with and without mixing. Calculations have been made for up to four passes. The primary interest was to find convenient calculation models and to compare them with other published models. One assumption was that no longitudinal heat conduction occurred in the heat exchanger wall. In the second part calculations were performed for the case when longitudinal heat conduction in the plates was included. In the third part results from the calculations without and with longitudinal heat conduction in the wall were compared with test results from a small crossflow heat exchanger. The test results showed good agreement with the results from the calculations with longitudinal heat conduction. The following conclusions can be made. When calculations are made on crossflow heat exchangers working with air the longitudinal heat conduction normally has to be taken into consideration. The error that occurs is easy to assess by means of the results given in this report. The conduction always lowers the effectiveness of the heat exchanger, and sometimes a better efficiency can be obtained if the plate between the two airstreams has a <u>low</u> conductivity.			
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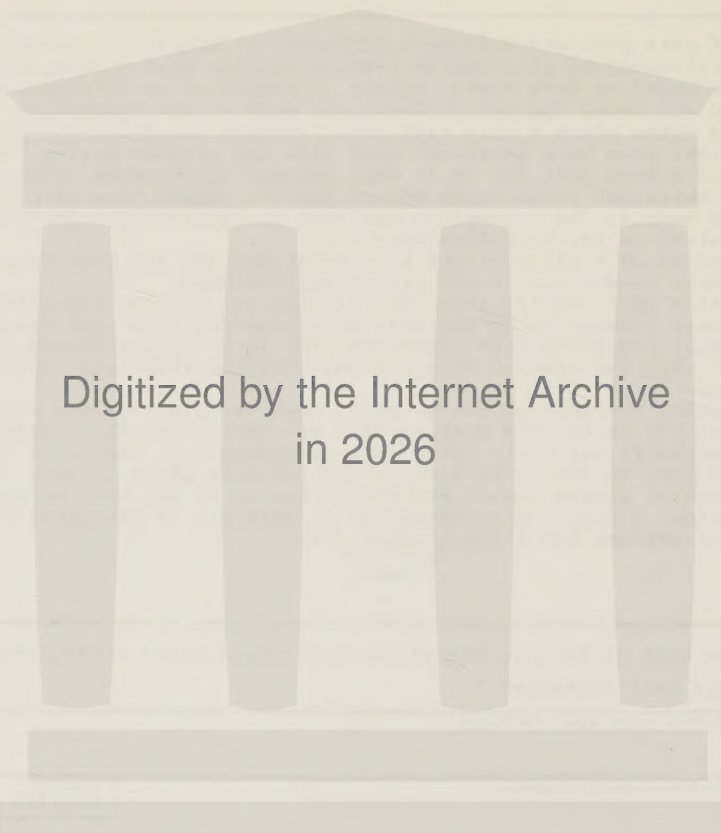
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SYMBOLS

A	heat transfer area	m^2
$A_{i,j}$	heat transfer area in element i,j	m^2
c	velocity	$\frac{m}{s}$
$C = \dot{m}C_p$	capacity	$\frac{J}{sK}$
C_p	specific isobaric heat capacity	$\frac{J}{kgK}$
$d = \frac{4S}{P}$	hydraulic diameter	m
D	diameter	m
f_g	form factor, Eq (8.4)	
$G = \rho \cdot c$	mass velocity	$\frac{kg}{m^2 s}$
h	heat transfer coefficient	$\frac{W}{m^2 K}$
$hNTU = \frac{h \cdot A}{C}$	number of transfer units based on the heat transfer coefficient, Eqs (7.32) and (7.33)	
i	specific enthalpy	$\frac{J}{kgK}$
k	thermal conductivity	$\frac{W}{mK}$
$K_c = \frac{kL\delta}{C_c \ell}$	longitudinal heat conduction parameter, Eq (7.36)	
$K_h = \frac{k\ell\delta}{C_h L}$	Eq (7.35)	
L, ℓ	length of heat exchanger in x - and y -direction, respectively, Fig 2a, b and 29	m
$\dot{m}$	mass flow rate	$\frac{kg}{s}$
n	number of passes	
M	number of elements in the x -direction	
N	number of elements in the y -direction	

$NTU = \frac{U \cdot A}{C_{\min}}$	number of transfer units, Eq (3.2)	
$NTU_c = \frac{U \cdot A}{C_c}$	Eq (3.3b)	
$NTU_h = \frac{U \cdot A}{C_h}$	Eq (3.3a)	
$Nu = \frac{hd}{k}$	Nusselt number	
p	pressure	Pa
P	perimeter	m
$Pr = \frac{C_p \mu}{k}$	Prandtl number	
q	heat flow rate	W
$q'' = \frac{dq}{dA}$	heat flow intensity	$\frac{W}{m^2}$
r	latent heat of evaporation	$\frac{J}{kg}$
R	gas constant	$\frac{J}{kgK}$
$Re = \frac{\rho c d}{\mu}$	Reynolds number	
S	cross sectional area	m^2
t	temperature of cold fluid	K
T	temperature (chapter 7, hot fluid)	K
$\bar{T}$	average temperature, Eq (8.32)	K
ΔT_m	mean temperature difference, Eqs (4.10), (4.33), (4.51)	K
U	overall heat transfer coefficient	$\frac{W}{m^2 K}$
$\dot{V}$	volume flow rate	$\frac{m^3}{s}$
x	coordinate in the flow direction of the hot fluid, Fig 2a, 2b and 29	m
$x' = \frac{x}{L}$	dimensionless coordinate in the flow direction of the hot fluid	
y	coordinate in the flow direction of the cold fluid	m

$y' = \frac{y}{x}$	dimensionless coordinate in the flow direction of the cold fluid	
$Y_c = \frac{k\delta}{U\lambda^2}$	Eq (7.56)	
$Y_h = \frac{k\delta}{UL^2}$	Eq (7.55)	
δ	thickness of heat exchanger wall	m
ϵ	effectiveness of the heat exchanger, Eq (3.7)	
$\Delta\epsilon_{rel}$	Eq (7.58)	
η	temperature efficiency, Eqs (3.5), (3.6)	
θ	wall temperature	K
ξ	friction factor, Eq (8.6)	
ρ	density	$\frac{kg}{m^3}$
ϕ	dimensionless temperature, Eqs (4.14) and (4.79)	
ϕ	relative humidity	
Φ	Eq (3.1)	
μ	dynamic viscosity	$\frac{kg}{sm}$
$\omega = \frac{\rho_v}{\rho_a}$	humidity ratio, Eq (8.18)	

Subscripts

a	dry air
c	cold side
dyn	dynamic
e	element
g	saturation
h	hot side
i	inner, Eqs (8.3) and (8.4)
i,j	subscript of the temperature arrays

in inlet of the heat exchanger
o outer, Eqs (8.3) and (8.4)
o maximum length, Eq (3.8)
p pass
v vapor in air
1 inlet of heat exchanger or element
2 outlet of heat exchanger or element

The use of heat exchangers has in recent years become more common in air-to-air systems to reduce energy consumption, for example in houses, factories, drying processes etc.. One kind of heat exchanger which is used in these processes is the plate cross-flow heat exchanger. The crossflow heat exchanger can be of single pass type or of multipass type, Fig 1a, b.

The first part of this report presents different mathematical models for calculating the effectiveness of thin walled plate crossflow heat exchangers operating with gases on both sides, either single or in multipass arrangements. In all cases the calculations are made on the assumption that no mixing takes place within each pass. Between the different passes calculations have been performed both with and without mixing. Calculations have been made for up to four passes. In this first part of the report the primary interest was to find as convenient calculation models as possible and to compare them with other published models. One important assumption was that no longitudinal heat conduction occurred in the heat exchanger wall.

In the second part of this report the same calculations were performed for the case when longitudinal heat conduction in the plates was included.

In the third part of this work, results from the calculations without and with longitudinal heat conduction in the wall were compared with test results from a small crossflow heat exchanger. The test results showed good agreement with the results from the calculations with longitudinal heat conduction. The error analysis regarding heat balance also showed good agreement with the test results as soon as steady state was obtained.

Some calculations (section 3.5) have been performed to show the influence of non-uniform inlet temperature and mass velocity profiles.

The overall heat transfer coefficient U for a plate can be found from the formula

$$\frac{1}{U} = \frac{1}{h_c} + \frac{1}{h_h} + \frac{\delta}{k} \quad \dots\dots\dots (1.1)$$

The values of h_c and h_h are in the range of 20 to 40 $\frac{W}{m^2 K}$ for

gas-to-gas heat exchangers and δ is approximately 10^{-3} m.

For a heat exchanger made from metal k is in the range of 100 $\frac{W}{mK}$, and for plastics approximately 1 $\frac{W}{mK}$. The conduction

resistance will then be for metal $\frac{\delta}{k} = 10^{-5} \frac{m^2 K}{W}$ and for plastics $\frac{\delta}{k} = 10^{-3} \frac{m^2 K}{W}$. Compared with the resistance due to transfer convection ($\frac{1}{h} = \frac{1}{20} = 0.05$ to $\frac{1}{40} = 0.025$) this value is thus small so for the present applications the following approximation in chapter 7 has been made

$$\frac{1}{U} = \frac{1}{h_c} + \frac{1}{h_h} \quad \dots\dots\dots (1.2)$$

The problem of calculating the effectiveness of crossflow heat exchangers has been treated in many papers. Some of the most important papers will be mentioned here.

2.1

Single pass

In 1911 Nusselt [1] published a paper on the temperature profiles and effectiveness of a single pass crossflow heat exchanger. The partial differential equation of Nusselt [1] is derived as a special case, Eq (7.72) in section 7.12 below. The expressions that he derived were in the form of integrals and were rather complicated to use. In 1930 he published another paper [2], which gives the solution as a series and it was easier to use. Another series solution was given by Binnie and Poole [3] in 1937.

The expressions given in [3] converge faster than the series solution given by Nusselt [2]. In 1954 Mason [4] presented another solution. The partial differential equation of Mason [4] is derived as a special case, Eq (7.75), in section 7.12 below. He used the Laplace transform on the governing equations, and derived an infinite series which converges very rapidly. Later Baclic [5] showed that the solution given by Mason [4] was the same as the one given by Nusselt [1, 2]. Baclic also pointed out that the difference in convergence depended on the order in which the terms in the series appeared. He also showed that by rearranging the expression given by Mason the convergence became faster for the case when the capacities on both sides were about the same.

The five references mentioned above treated the problems of effectiveness and temperature profiles in single pass crossflow heat exchangers for the case of uniform mass velocity ($G = \rho \cdot c$) and temperature profiles at the inlet of the heat exchanger. These references also treated the case of no mixing occurring within the heat exchanger. In 1974 M Bentwich and A Tal [6] presented a paper in which they treated the problem of calculating the heat flow intensity q'' through the heat exchanger

at different locations. They also pointed out the problem of non-uniform mass flow through the heat exchanger.

2.2

Multipass arrangement

The calculation of multipass arrangements of crossflow heat exchangers has been dealt with in many papers. One of the more comprehensive papers was published by R A Stevens, J Fernandez and J R Woolf [7]. They treated the different types of heat exchanger arrangements with difference equations in 1957 and divided each pass of the heat exchanger into 20×20 elements and used a cyclical reiteration process for the countercurrent case. In each element they used the formula according to the NTU-concept for the calculation of the effectiveness of this element for the cases where no, one or two fluids are mixed in the element. By using this technique they could calculate each pass of the heat exchanger. Combining this with the two cases of mixing and no mixing between passes and co-current and countercurrent arrangements, they solved numerically all cases up to three pass heat exchangers. The range of NTU-values was from 0 to 7. The results were presented in some charts showing the effectiveness for the crossflow heat exchanger compared with a countercurrent or cocurrent heat exchanger, depending on whether the arrangement for the crossflow heat exchanger was countercurrent or cocurrent. The authors claimed that the fourth figure in the effectiveness is significant. The calculations were made under the assumption that the mass velocity profiles at the inlets were uniform in space. In 1957 Sahlberg [8] published a paper on the same subject but he treated the problem differently. Part of his solution is well known from Nusselt and others. He worked, however, also with multipass arrangements and took into consideration the non-uniform temperature profiles that will arise between the different passes. He solved this problem by coordinate transformation and superposition. The solution is rather mathematical and not easy to use.

In 1976 T Ishimaru, N Kokubo and R Izumi [9] published a paper in which they used the Laplace transform in such a way that they could derive the mass velocity profile in the heat exchanger to get a uniform outlet temperature on the cold side. Their paper is one of the few which treats the problem of non-uniform mass velocity. In 1976 Chiou [10] also presented a paper on non-uniform massflow distribution in crossflow heat exchangers. He used a finite difference method. Chiou treated the case of a single crossflow heat exchanger and made extensive calculations on the effect on the effectiveness by non-uniform mass velocity profile. In 1976 he also presented a paper [11] on the influence of longitudinal heat conduction in the wall of a single pass crossflow heat exchanger. No experimental results were given.

This literature review does not cover all different aspects on crossflow heat exchangers but it is intended to give a broad outline of what has happened in the last century.

2.4 Different aspects of calculations of crossflow heat exchangers

The works done earlier by other investigators show a tendency to be more a question of getting as many significant figures as possible for the mathematical model [5, 7, 8]. An example of this is [7] where the authors claim that four figures are significant for their model. A more relevant question would be: How many figures in the effectiveness are relevant in the model if one takes into consideration factors that have been left out, e.g. longitudinal conduction and non-uniform inlet temperature and mass velocity profiles? It will be shown below that by means of a more complex model a much better agreement between theory and experiment is obtained.

The aim of the present work is to find models which can be used for single and multipass arrangements. The models will be more or less complex. Models concerning no mixing in passes will be presented, and measurements on a single crossflow heat

exchanger will be compared with the calculated results.

In the following chapters there will be some statements on the accuracy of the methods. It must be clear that the different statements are on the accuracy of the mathematical model and not of the actual working heat exchanger.

This chapter will deal with the problem of the temperature profiles in crossflow heat exchangers. The effect of non-uniform mass velocity and temperature profiles will be taken into consideration. In this work a step-by-step method has been used.

3.1 Overall calculations without longitudinal heat conduction

All the theory is based on the effectiveness ϵ , number of transfer units NTU, and the capacity ratio $C_{\min}/C_{\max}$. For a crossflow heat exchanger the effectiveness can be calculated in different ways. The solution given by John L Mason [4] in 1954 is however a solution which converges very rapidly and which treats a single pass crossflow heat exchanger without mixing. The following expressions are given in [4]

$$\phi = \frac{1}{NTU_h \cdot NTU_c} \sum_{n=0}^{\infty} R_n(NTU_h) R_n(NTU_c) \quad \dots\dots\dots (5.1)$$

$$NTU = \frac{UA}{C_{\min}} \quad \dots\dots\dots (3.2)$$

where the two NTU-values

$$NTU_h = \frac{UA}{\dot{m}_h \cdot C_{p_h}} = \frac{UA}{C_h} \quad \dots\dots\dots (3.3a)$$

$$NTU_c = \frac{UA}{\dot{m}_c \cdot C_{p_c}} = \frac{UA}{C_c} \quad \dots\dots\dots (3.3b)$$

$$R_n(NTU) = 1 - e^{-NTU} \sum_{i=0}^n \frac{NTU^i}{i!} \quad \dots\dots\dots (3.4)$$

The temperature efficiencies

$$\eta_h = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} = NTU_h \phi \quad \dots\dots\dots (3.5)$$

$$\eta_c = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = NTU_c \phi \quad \dots\dots\dots (3.6)$$

Another common way of determining the heat exchanger performance is by means of the effectiveness [12]

$$\left. \begin{aligned} \epsilon &= \frac{q}{C_{\min}(T_{h1} - T_{c1})} \\ \text{where } C_{\min} &= \text{Min}((\dot{m}C_p)_c, (\dot{m}C_p)_h) \text{ and} \\ q &= C_h(T_{h1} - T_{h2}) = C_c(T_{c2} - T_{c1}) \end{aligned} \right\} \dots\dots\dots (3.7)$$

Thus ϵ is the ratio of the heat flow rate q , which is transferred through the heat exchanger, and the maximum heat flow rate which could be transferred through a countercurrent heat exchanger with infinite area. The temperature efficiency on the other hand is only related to temperatures. If the NTU-values on the cold and hot side are known and the inlet temperatures are known, the outlet temperatures can be calculated from Eqs (3.1) - (3.6).

If no mixing takes place in the single pass crossflow heat exchanger Nusselt [1] showed that the temperature distribution on the hot side is given by

$$T_h = T_{h1} e^{-\frac{UA}{C_h} \frac{y}{y_o}} \left[1 + \frac{U^2 A^2}{C_h C_c} \frac{xy}{x_o y_o} \int_0^{\sqrt{z}} - \frac{i J_1(2i\sqrt{z})}{\sqrt{z}} e^{-\frac{C_h y_o z}{UAy}} dz \right] \dots (3.8)$$

and on the cold side:

$$T_c = \frac{T_{h1} e^{-\frac{UA}{C_h} \frac{y}{y_o}} \frac{U^2 A^2}{C_h C_c} \frac{xy}{x_o y_o} \int_0^{\sqrt{z}} J_0(2i\sqrt{z}) e^{-\frac{C_h y_o z}{UAy}} dz}{\frac{UA}{C_h} \frac{y}{y_o}} \dots\dots\dots (3.9)$$

where J_0 and J_1 are the Bessel functions of order zero and one.

These two expressions are obtained if the inlet mass velocities and temperatures are assumed to be uniform.

Nusselt [2] has given an alternative series solution. In 1957 Sahlberg [8] used the theory by Nusselt to calculate the effectiveness and temperature distribution in one to four pass crossflow heat exchangers in various arrangements. He also treated the cases with non-uniform mass velocities and temperatures. Sahlberg pointed out the problem of convergence for NTU-values greater than 4. Because of the rather long calculation times when one uses Sahlberg's method, for NTU-values of about 3 and greater, the question arises: Can a simpler method of calculation be used instead?

3.2

Background

A step-by-step method based on the expressions by Mason [4] is used. In Fig 2a the definitions of capacities and temperature are given. By using Eqs (3.5) - (3.6) the result for a crossflow heat exchanger will be:

$$T_{h_2} = T_{h_1} - \eta_h(T_{h_1} - T_{c_1}) \quad \dots\dots\dots(3.10)$$

$$T_{c_2} = T_{c_1} + \eta_c(T_{h_1} - T_{c_1}) \quad \dots\dots\dots(3.11)$$

The calculation is based on uniform inlet mass velocities and temperatures but it can also be used for non-uniform inlet mass velocities and temperatures by the following difference method given in the following section. This method is also used in [7] for uniform mass velocities.

3.3 Non-uniform inlet mass velocities and temperatures

In Fig 2b the crossflow heat exchanger has been divided into elements (cells) in which inlet temperatures and mass velocities are constant. This means that if the flow profiles into the heat exchanger have any arbitrary shape they can be divided into elements (cells) with constant inlet mass velocities and temperatures, which can be calculated in the following way:

$$C_{he}(j) = \frac{1}{L_e} \int_0^{x_e} C_h(y) dy \quad \dots\dots\dots (3.12)$$

$$C_{ce}(i) = \frac{1}{L_e} \int_0^{L_e} C_c(x) dx \quad \dots\dots\dots (3.13)$$

$$T_{h0,j} = \frac{1}{\dot{m}_{he0,j}} \int \dot{m}_{he0,j} T_h(y) dy \quad \dots\dots\dots (3.14)$$

$$T_{ci,0} = \frac{1}{\dot{m}_{cei,0}} \int \dot{m}_{cei,0} T_c(x) dx \quad \dots\dots\dots (3.15)$$

Four vectors for the inlet conditions can now be formed:

$$\left. \begin{aligned} C_h &= (C_{he}(1), C_{he}(2), \dots, C_{he}(N)) \\ C_c &= (C_{ce}(1), C_{ce}(2), \dots, C_{ce}(M)) \end{aligned} \right\} \dots\dots\dots (3.16)$$

$$\left. \begin{aligned} T_h &= (T_{h0,1}, T_{h0,2}, \dots, T_{h0,N}) \\ T_c &= (T_{c1,0}, T_{c2,0}, \dots, T_{cM,0}) \end{aligned} \right\} \dots\dots\dots (3.17)$$

In the following calculation $N = M$ but in general N and M can be any numbers. Eqs (3.1) through (3.6) give the possibility of calculating the temperature efficiency on the hot and cold side for the heat exchanger element (cell) (i,j) , which can be seen as a small heat exchanger with the inlet conditions $C_{he}(j)$, $C_{ce}(i)$, $T_h(i-1,j)$, $T_c(i,j-1)$, since the temperatures on the cold and hot side can be represented by two matrices $T_h(M,N)$ and $T_c(M,N)$. For an arbitrary cell (i,j) the result will be (Eqs (3.10) and (3.11))

$$T_{ci,j} = T_{ci,j-1} + n_{ci,j} (T_{hi-1,j} - T_{ci,j-1}) \quad \dots\dots\dots (3.18)$$

$$T_{h_{i,j}} = T_{h_{i-1,j}} - \eta_{h_{i,j}} (T_{h_{i-1,j}} - T_{c_{i,j-1}}) \dots\dots\dots(3.19)$$

The calculation of the temperature efficiency for the element on the cold side $\eta_{c_{i,j}}$ and on the hot side $\eta_{h_{i,j}}$ will now be performed. This calculation is easily done, because $C_{he}(j)$ and $C_{ce}(i)$ are given.

The heat transfer area for element (cell) (i,j) equals

$$A_e = A_{i,j} = \frac{A}{N \cdot M} \dots\dots\dots(3.20)$$

where A is the total heat transfer area of one plate.

The overall heat transfer coefficient is assumed to be known. This means that $NTU_{ce_{i,j}}$ and $NTU_{he_{i,j}}$ are known. Thus

$$NTU_{ce_{i,j}} = \frac{U \cdot A}{C_{ce}(i) \cdot M \cdot N} \dots\dots\dots(3.21)$$

$$NTU_{he_{i,j}} = \frac{U \cdot A}{C_{he}(j) \cdot M \cdot N} \dots\dots\dots(3.22)$$

By means of Eqs (3.1) through (3.6) the temperature efficiency can now be calculated in the element (cell).

From Eqs (3.18) through (3.19) the temperature profiles on the cold and hot sides can be calculated within the heat exchanger. If the inlet temperatures of the heat exchanger are $T_{c_{i,0}}$ and $T_{h_{0,j}}$, then the temperature in any element (cell) (i,j) can be written:

$$T_{c_{i,j}} = T_{c_{i,0}} + \sum_{j=1}^j \eta_{c_{i,j}} (T_{h_{i-1,j}} - T_{c_{i,j-1}}) \dots\dots\dots(3.23)$$

$$T_{h_{i,j}} = T_{h_{0,j}} - \sum_{i=1}^i \eta_{h_{i,j}} (T_{h_{i-1,j}} - T_{c_{i,j-1}}) \dots\dots\dots(3.24)$$

By use of Eqs (3.23) through (3.24) or Eqs (3.18) through (3.19) the temperature profiles of the heat exchanger can be obtained.

The equations (3.23) and (3.24) or (3.18) and (3.19) can be solved for all i 's and j 's, by first calculating all j 's for constant i and then calculating the next i , or first calculating all i 's for constant j and then calculating the next j . The accuracy of the method depends on NTU_h , NTU_c , the number of elements (cells) and inlet temperature and mass velocity profiles. That the accuracy depends on the temperatures and mass velocity profiles can be explained by the fact that the correct profiles have to be transformed into step functions. At each element outlet the cold and hot temperatures are considered to be constant, which is approximately correct. By dividing the heat exchanger into several small heat exchangers the accuracy can be increased. This problem will be dealt with in chapter 4.

The important advantages of the method of section 3.3 are

1. Any temperature or mass velocity profile can be transformed into step functions.
2. By dividing the heat exchanger into small parts of arbitrary size there will not be any convergence problem.
3. The calculations can easily be done on a computer.
4. The calculations will not be more difficult because of difficult inlet conditions.
5. Interesting parts of the heat exchanger can easily be more accurately calculated.
6. Isotherms can be detected.

One disadvantage can, however, be seen in Fig 2b. The profiles plotted are the profiles given by the average temperatures in each element, and therefore the boundary temperatures (side walls parallel to flow) are not plotted. This problem can easily be solved by extrapolating half the element width with a polynomial in order to get these boundary temperatures.

The program has been made in Standard FORTRAN and the aim of the work has been to find a fast way of calculating the temperatures and not at this stage to get the best accuracy. Therefore a program with integer temperatures between 10 000 and 0 has been used, which gives only a slight error. To exemplify this, Sahlberg's method [8] and the present method have been compared for one case, $NTU_c = 4.0$ and $NTU_h = 8.0$ (20×20 elements). The two methods give the following result on the cold side with an inlet temperature of 0 and on the hot side with an inlet temperature of 10 000.

Table 3.5.1 Cold side temperature

$NTU_c = 4.0$

NTU_h	Sahlberg ⁺	Present method 20×20 elements	Diff	Diff % Percent of the Sahlberg value
0.2	9656	9628	- 28	- 0.29
0.6	9255	9225	- 30	- 0.32
1.0	8766	8736	- 30	- 0.34
1.4	8211	8182	- 29	- 0.35
1.8	7610	7584	- 26	- 0.34
2.2	6985	6963	- 22	- 0.32
2.6	6354	6337	- 17	- 0.27
3.0	5731	5717	- 14	- 0.24
3.4	5129	5119	- 10	- 0.20
3.8	4556	4550	- 6	- 0.13
4.2		4017		
4.6		3523		
5.0		3072		
5.4		2663		
5.8		2297		
6.2		1970		
6.6		1683		
7.0		1431		
7.4		1211		
7.8		1020		

⁺from [8]

From Table 3.5.1 it can be seen that the difference between the present method and Sahlberg's method is very small and by using real (non-integer) variables the difference would be even smaller. Table 3.5.1 also shows that in the present method there are no convergence problems for high NTU-values as in the case of the Sahlberg method. In the following pages the results of the method will be represented by plots of the temperature profiles under different conditions. The small error in temperature has little practical importance compared with the error that can occur due to the NTU-numbers because of the uncertainty in the heat transfer coefficients. In Fig 3 and 4 the temperature profiles for the hot and cold sides are plotted for $NTU_c = 4.0$, $NTU_h = 8.0$, and for uniform inlet mass velocity and temperature profiles. The number of elements (cells) was 20×20 . In Fig 5 and 6 all temperature values have been reduced by 5000 and all negative values which appear for temperatures under 5000 are replaced by the value 0. For that reason the breakpoints between the straight lines and the curved lines represent a temperature of 5000 (0.5). This method gives the possibility of plotting isotherms in the heat exchanger and temperatures above the value of the given isotherm. In Fig 5 the hot side is plotted under the same conditions as in Fig 3 and Fig 6 (cold side) corresponds to Fig 4.

Fig 7 shows the hot side temperature profile in a heat exchanger, with the following conditions: $NTU_c = 3.6$, $NTU_h = 3.6$, and uniform mass velocity profiles on the hot and cold sides. The hot inlet temperature equals 10 000, and the cold inlet temperature equals 0. The heat exchanger is here divided into a matrix of 9×9 elements. This gives an element (cell) (i,j) NTU-number of 0.4. Fig 8 shows the hot side temperature profile of the same heat exchanger under the same flow conditions. The only difference is that the matrix now consists of 36×36 elements (cells), which gives an element (cell) NTU-number on the hot and cold sides of 0.1. From Fig 7-8 it can be seen that a reasonably good temperature profile can be obtained with as few elements as 9×9 .

To demonstrate the usefulness of the method a more complex inlet flow pattern was assumed. The following element inlet mass flow rates and temperatures were chosen

$$\dot{m}_{he_{0,j}} = (0.01 \cdot j + 0.7)$$

$$\dot{m}_{ce_{i,0}} = (0.02 \cdot i + 0.4)$$

$$T_{h_{0,j}} = (0.02 \cdot j + 0.4) \cdot 10\ 000$$

$$T_{c_{i,0}} = (0.02 \cdot i) \cdot 10\ 000$$

The number of elements (cells) is 30×30 . The element NTU values have been divided into a constant factor and a varying factor on both sides depending on where the cell (i,j) is located

$$NTU_{he_{i,j}} = \frac{A_{i,j} \cdot U}{C_p} \cdot \frac{1}{\dot{m}_{he_{0,j}}} \dots\dots\dots (3.25)$$

$$NTU_{ce_{i,j}} = \frac{A_{i,j} \cdot U}{C_p} \cdot \frac{1}{\dot{m}_{ce_{i,0}}} \dots\dots\dots (3.26)$$

Fig 9 and 10 show temperature profiles for this complex flow pattern in the heat exchanger. Fig 9 shows the temperature profile for the hot side of the heat exchanger, and it can be seen that the temperature profile is rather complex. The same thing can be said about Fig 10 for the cold side of the heat exchanger. The most interesting parts are the lower left corner of Fig 9-10 because here heat transfer occurs from the "cold" to the "hot" side. This phenomenon can also be seen in Fig 11-12. Fig 11 is the plot of the hot side with an isotherm at 4000 and Fig 12 is a plot of the cold side with an isotherm at 5000. From these two figures it can also be seen that there is a heat transfer from the "cold" to the "hot" side in the lower left corner because the hot side increases in temperature in the flow direction and the cold side decreases in the flow direction.

The method of this chapter has also been used to calculate the change in effectiveness ϵ , due to non-uniform inlet temperature and mass velocity profiles, with the assumption that the mean values at the inlets were the same in all cases. The calculations have been performed for a single pass heat exchanger divided into 21×21 elements and in all cases

$\frac{A_{i,j} \cdot U}{C_p} = 0.1$. The inlet temperature and mass velocity profiles were varied linearly.

The average temperatures at the inlet and outlet of the heat exchanger can be calculated in the following way (compare Eq (8.32) below) for $C_p = \text{constant}$ and for a heat exchanger divided into $N \times N$ elements

$$\bar{T}_{c1} = \frac{\sum_{i=1}^N T_{c_{i,0}} \cdot \dot{m}_{ce_{i,0}}}{\sum_{i=1}^N \dot{m}_{ce_{i,0}}} \dots\dots\dots (3.27)$$

$$\bar{T}_{h1} = \frac{\sum_{i=1}^N T_{h_{0,i}} \cdot \dot{m}_{he_{0,i}}}{\sum_{i=1}^N \dot{m}_{he_{0,i}}} \dots\dots\dots (3.28)$$

$$\bar{T}_{c2} = \frac{\sum_{i=1}^N T_{c_{i,N}} \cdot \dot{m}_{ce_{i,0}}}{\sum_{i=1}^N \dot{m}_{he_{i,0}}} \dots\dots\dots (3.29)$$

$$\bar{T}_{h2} = \frac{\sum_{i=1}^N T_{h_{N,i}} \cdot \dot{m}_{he_{0,i}}}{\sum_{i=1}^N \dot{m}_{he_{0,i}}} \dots\dots\dots (3.30)$$

The effectiveness {Eqs (3.7) and (8.33)} will then be for

$$C_{\min} = C_c$$

$$\epsilon = \frac{\bar{T}_{c_2} - \bar{T}_{c_1}}{\bar{T}_{h_1} - \bar{T}_{c_1}} \dots\dots\dots(3.31)$$

The results of the calculations are given in Table 3.5.2.

Table 3.5.2 The influence on effectiveness by non-uniform inlet temperature and mass velocity profiles.

Case	$T_{h0,1}$	$T_{h0,21}$	$T_{c1,0}$	$T_{c21,0}$	$\dot{m}_{he0,1}$	$\dot{m}_{he0,21}$	$\dot{m}_{ce1,0}$	$\dot{m}_{ce21,0}$	ε
1	1.0	1.0	0.0	0.0	1.0	1.0	1.0	1.0	0.622
2	0.5	1.5	0.0	0.0	1.0	1.0	1.0	1.0	0.665
3	1.5	0.5	0.0	0.0	1.0	1.0	1.0	1.0	0.580
4	1.0	1.0	0.5	-0.5	1.0	1.0	1.0	1.0	0.665
5	1.0	1.0	-0.5	0.5	1.0	1.0	1.0	1.0	0.580
6	0.5	1.5	0.5	-0.5	1.0	1.0	1.0	1.0	0.707
7	1.5	0.5	-0.5	0.5	1.0	1.0	1.0	1.0	0.538
8	1.0	1.0	0.0	0.0	0.5	1.5	1.0	1.0	0.611
9	1.0	1.0	0.0	0.0	1.5	0.5	1.0	1.0	0.611
10	1.0	1.0	0.0	0.0	1.0	1.0	0.5	1.5	0.611
11	1.0	1.0	0.0	0.0	1.0	1.0	1.5	0.5	0.611

Table 3.5.2 The influence on effectiveness by non-uniform inlet temperature and mass velocity profiles (continued).

Case	$T_{h0,1}$	$T_{h0,21}$	$T_{c1,0}$	$T_{c21,0}$	$\dot{m}_{he0,1}$	$\dot{m}_{he0,21}$	$\dot{m}_{ce1,0}$	$\dot{m}_{ce21,0}$	ϵ
12	0.5	1.5	0.5	-0.5	0.5	1.5	1.0	1.0	0.668
13	0.5	1.5	0.5	-0.5	1.5	0.5	1.0	1.0	0.721
14	0.5	1.5	0.5	-0.5	1.0	1.0	0.5	1.5	0.668
15	0.5	1.5	0.5	-0.5	1.0	1.0	1.5	0.5	0.721
16	1.5	0.5	-0.5	0.5	0.5	1.5	1.0	1.0	0.542
17	1.5	0.5	-0.5	0.5	1.5	0.5	1.0	1.0	0.520
18	1.5	0.5	-0.5	0.5	1.0	1.0	0.5	1.5	0.542
19	1.5	0.5	-0.5	0.5	1.0	1.0	1.5	0.5	0.520
20	0.75	1.25	0.0	0.0	1.0	1.0	1.0	1.0	0.643
21	1.25	0.75	0.0	0.0	1.0	1.0	1.0	1.0	0.601
22	1.0	1.0	0.25	-0.25	1.0	1.0	1.0	1.0	0.643
23	1.0	1.0	-0.25	0.25	1.0	1.0	1.0	1.0	0.601
24	0.75	1.25	0.25	-0.25	1.0	1.0	1.0	1.0	0.665
25	1.25	0.75	-0.25	0.25	1.0	1.0	1.0	1.0	0.580

From Table 3.5.2 it can be seen that the effectiveness can be affected to a high degree by non-uniform inlet profiles. The influence of inlet temperature profiles is particularly important and it can also be seen that the inlet temperature profiles can cause an increase or decrease in effectiveness of the heat exchanger. The effect due to inlet mass velocity is not so pronounced for the cases considered. The influence, however, can be greater at higher NTU-values.

From the given examples and Fig 3-12, it can be seen that the method gives a powerful tool for computing temperature profiles in a crossflow heat exchanger.

The calculation method used in section 3.3 through 3.5 is the same as method 6 in section 4.4.5 below.

4. CALCULATIONS ON SINGLE PASS CROSSFLOW HEAT EXCHANGERS

4.1 Methods of calculations

Six different ways of calculating numerically the effectiveness of crossflow heat exchangers will be described and will be compared with calculations made by means of Mason [4] formula given in section 3.1. They all have the common feature that they also give the temperature profiles in the heat exchanger. From these six different ways the best methods will be selected so as to give the best accuracy. The merits of the different methods will also be discussed.

The inlet mass velocity and temperature profiles are assumed to be uniform and the overall heat transfer coefficient U and specific heat capacity C_p are also assumed to be constant in the heat exchanger.

4.2 Approach

Mason [4], Nusselt [1, 2], Sahlberg [8], and Baclic [5] use analytical methods for calculating the effectiveness ϵ of a crossflow heat exchanger, when the mass velocity is uniform in the heat exchanger and when the two dimensionless groups, the number of transfer units NTU, and the capacity ratio are assumed to be known [1, 2, 4, 5, 8]. In this chapter numerical methods will be used and the results will be compared with that obtained from Mason's formula.

4.3 Dimensionless groups

For the whole heat exchanger

$$NTU_c = \frac{U \cdot A}{C_c} \dots\dots\dots (4.1)$$

$$\frac{C_c}{C_h} = \frac{\dot{m}_c C_{p_c}}{\dot{m}_h C_{p_h}} \dots\dots\dots (4.2)$$

If the heat exchanger is divided into $N \times N$ elements (cells), Fig 13, and if the mass velocities are uniform and the overall heat transfer coefficient U and the specific heat capacity C_p are constant on each side, the NTU-number for each element (cell) can be calculated

$$NTU_{ce} = \frac{U \cdot \frac{A}{N \cdot N}}{\frac{C}{N}} = \frac{U \cdot A}{N(C_c)} \dots\dots\dots (4.3)$$

Thus

$$NTU_e = \frac{1}{N} NTU \dots\dots\dots (4.4)$$

$$C_e = \frac{C}{N} \dots\dots\dots (4.5)$$

$$\frac{C_{ce}}{C_{he}} = \frac{C_c}{C_h} \dots\dots\dots (4.6)$$

Thus NTU_e is related to NTU for the heat exchanger by Eq (4.4) and the capacity ratio is the same for the element as for the whole heat exchanger according to Eq (4.6).

For the element

$$NTU_{ce} = \frac{A_e U}{C_{ce}} \dots\dots\dots (4.7)$$

4.4 One element

4.4.1 Method 1

$$q_e = C_{ce}(T_{ce2} - T_{ce1}) \dots\dots\dots (4.8)$$

$$q_e = -C_{he}(T_{he2} - T_{he1}) = C_{he}(T_{he1} - T_{he2}) \dots\dots\dots (4.9)$$

where 1 means element inlet and 2 means element outlet.

Assume that the mean temperature difference is approximately

$$\Delta T_{me} = T_{he2} - T_{ce2} \dots\dots\dots (4.10)$$

$$q_e = UA_e \Delta T_{me} = UA_e(T_{he2} - T_{ce2}) \dots\dots\dots (4.11)$$

These four equations are transformed into dimensionless form in the following way

$$\phi_{c_2} = a\phi_{c_1} + b\phi_{h_2} \dots\dots\dots(4.12)$$

$$\phi_{h_2} = e\phi_{h_1} + f\phi_{c_2} \dots\dots\dots(4.13)$$

where

$$\left. \begin{aligned} \phi_{c_i} &= \frac{T_{ce_i} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} \\ \phi_{h_i} &= \frac{T_{he_i} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} \end{aligned} \right\} \dots\dots\dots(4.14)$$

where $T_{h_{in}}$ and $T_{c_{in}}$ are the uniform inlet temperatures of the heat exchangers.

Eqs (4.8) through (4.11) give

$$UA_e(T_{he_2} - T_{ce_2}) = C_{ce}(T_{ce_2} - T_{ce_1}) = C_{he}(T_{he_1} - T_{he_2}) \dots(4.15)$$

$$T_{ce_2} - T_{ce_1} = \frac{UA_e}{C_{ce}} (T_{he_2} - T_{ce_2}) \dots\dots\dots(4.16)$$

$$T_{he_2} - T_{he_1} = - \frac{UA_e}{C_{he}} (T_{he_2} - T_{ce_2}) \dots\dots\dots(4.17)$$

Eq (4.16) gives

$$T_{ce_2} - T_{ce_1} = \frac{UA_e}{C_{ce}} T_{he_2} - \frac{UA_e}{C_{ce}} T_{ce_2} \dots\dots\dots(4.18)$$

$$T_{ce_2} (1 + \frac{UA_e}{C_{ce}}) = T_{ce_1} + \frac{UA_e}{C_{ce}} T_{he_2} \dots\dots\dots(4.19)$$

Subtract $T_{c_{in}} \left(1 + \frac{UA_e}{C_{ce}}\right)$ from both sides

$$\begin{aligned} (T_{ce_2} - T_{c_{in}}) \left(1 + \frac{UA_e}{C_{ce}}\right) &= T_{ce_1} - T_{c_{in}} + \\ &+ \frac{UA_e}{C_{ce}} (T_{he_2} - T_{c_{in}}) \dots\dots\dots (4.20) \end{aligned}$$

After division by $T_{h_{in}} - T_{c_{in}}$ we obtain

$$\begin{aligned} \frac{T_{ce_2} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} \left(1 + \frac{UA_e}{C_{ce}}\right) &= \\ &= \frac{T_{ce_1} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} + \frac{UA_e}{C_{ce}} \frac{T_{he_2} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} \dots\dots\dots (4.21) \end{aligned}$$

Eqs (4.21) and (4.14) give

$$\phi_{c_2} = a_1 \phi_{c_1} + b_1 \phi_{h_2} \dots\dots\dots (4.22)$$

, where

$$a_1 = \frac{1}{1 + \frac{UA_e}{C_{ce}}} \dots\dots\dots (4.23)$$

$$b_1 = \frac{\frac{UA_e}{C_{ce}}}{1 + \frac{UA_e}{C_{ce}}} \dots\dots\dots (4.24)$$

The same procedure is used for the hot side.

Eq (4.17) gives

$$T_{he_2} = T_{he_1} - \frac{UA_e}{C_{he}} (T_{he_2} - T_{ce_2}) \dots\dots\dots (4.25)$$

$$T_{he2} \left(1 + \frac{UA_e}{C_{he}}\right) = T_{he1} + \frac{UA_e}{C_{he}} T_{ce2} \quad \dots\dots\dots (4.26)$$

$$\begin{aligned} \frac{T_{he2} - T_{cin}}{T_{hin} - T_{cin}} \left(1 + \frac{UA_e}{C_{he}}\right) &= \frac{T_{he1} - T_{cin}}{T_{hin} - T_{cin}} + \\ &+ \frac{UA_e}{C_{he}} \frac{T_{ce2} - T_{cin}}{T_{hin} - T_{cin}} \quad \dots\dots\dots (4.27) \end{aligned}$$

$$\phi_{h2} = e_1 \phi_{h1} + f_1 \phi_{c2} \quad \dots\dots\dots (4.28)$$

, where

$$e_1 = \frac{1}{1 + \frac{UA_e}{C_{he}}} = \frac{1}{1 + \frac{UA_e}{C_{ce}} \cdot \frac{C_{ce}}{C_{he}}} \quad \dots\dots\dots (4.29)$$

$$f_1 = \frac{\frac{UA_e}{C_{he}}}{1 + \frac{UA_e}{C_{he}}} = \frac{\frac{UA_e}{C_{ce}} \cdot \frac{C_{ce}}{C_{he}}}{1 + \frac{UA_e}{C_{ce}} \cdot \frac{C_{ce}}{C_{he}}} \quad \dots\dots\dots (4.30)$$

4.4.2 Method 2

$$q_e = C_{ce}(T_{ce2} - T_{ce1}) \quad \dots\dots\dots (4.31)$$

$$q_e = -C_{he}(T_{he2} - T_{he1}) = C_{he}(T_{he1} - T_{he2}) \quad \dots\dots\dots (4.32)$$

Assume that the mean temperature difference is approximately

$$\Delta T_{me} = T_{he1} - T_{ce1} \quad \dots\dots\dots (4.33)$$

$$q_e = UA_e \Delta T_{me} = UA_e (T_{he1} - T_{ce1}) \quad \dots\dots\dots (4.34)$$

Eqs (4.31), (4.32) and (4.34) give

$$UA_e(T_{he1} - T_{ce1}) = C_{ce}(T_{ce2} - T_{ce1}) = C_{he}(T_{he1} - T_{he2}) \dots (4.35)$$

For cold side

$$T_{ce2} - T_{ce1} = \frac{UA_e}{C_{ce}}(T_{he1} - T_{ce1}) \dots (4.36)$$

$$T_{ce2} = \frac{UA_e}{C_{ce}} T_{he1} + (1 - \frac{UA_e}{C_{ce}}) T_{ce1} \dots (4.37)$$

Subtract $T_{cin} = \frac{UA_e}{C_{ce}} T_{cin} + (1 - \frac{UA_e}{C_{ce}}) T_{cin}$ and divide by

$$T_{hin} - T_{cin}$$

$$\frac{T_{ce2} - T_{cin}}{T_{hin} - T_{cin}} = \frac{UA_e}{C_{ce}} \frac{T_{he1} - T_{cin}}{T_{hin} - T_{cin}} + (1 - \frac{UA_e}{C_{ce}}) \frac{T_{ce1} - T_{cin}}{T_{hin} - T_{cin}} \dots (4.38)$$

Eqs (4.38) and (4.14) give

$$\phi_{c2} = a_2 \phi_{h1} + b_2 \phi_{c1} \dots (4.39)$$

, where

$$a_2 = \frac{UA_e}{C_{ce}} \dots (4.40)$$

$$b_2 = 1 - \frac{UA_e}{C_{ce}} \dots (4.41)$$

For hot side Eq (4.35) gives

$$UA_e(T_{he1} - T_{ce1}) = C_{he}(T_{he1} - T_{he2}) \dots (4.42)$$

$$T_{he1} - T_{he2} = \frac{UA_e}{C_{he}} (T_{he1} - T_{ce1}) \dots (4.43)$$

$$T_{he2} = (1 - \frac{UA_e}{C_{he}}) T_{he1} + \frac{UA_e}{C_{he}} T_{ce1} \dots (4.44)$$

Subtract $T_{c_{in}} = \frac{UA_e}{C_{he}} T_{c_{in}} + (1 - \frac{UA_e}{C_{he}}) T_{c_{in}}$ and divide by

$$T_{h_{in}} - T_{c_{in}}$$

$$\frac{T_{he_2} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} = (1 - \frac{UA_e}{C_{he}}) \frac{T_{he_1} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} + \frac{UA_e}{C_{he}} \frac{T_{ce_1} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} \dots\dots\dots (4.45)$$

Eqs (4.45) and (4.14) give

$$\phi_{h_2} = e_2 \phi_{h_1} + f_2 \phi_{c_1} \dots\dots\dots (4.46)$$

, where

$$e_2 = 1 - \frac{UA_e}{C_{ce}} \cdot \frac{C_{ce}}{C_{he}} \dots\dots\dots (4.47)$$

$$f_2 = \frac{UA_e}{C_{ce}} \cdot \frac{C_{ce}}{C_{he}} \dots\dots\dots (4.48)$$

4.4.3 Method 3

$$q_e = C_{ce} (T_{ce_2} - T_{ce_1}) \dots\dots\dots (4.49)$$

$$q_e = - C_{he} (T_{he_2} - T_{he_1}) = C_{he} (T_{he_1} - T_{he_2}) \dots\dots\dots (4.50)$$

Assume that the mean temperature difference is approximately

$$\Delta T_{me} = \frac{1}{2} \{ (T_{he_2} - T_{ce_2}) + (T_{he_1} - T_{ce_1}) \} \dots\dots\dots (4.51)$$

which thus is the average of method 1 and 2

$$q_e = UA_e \Delta T_{me} = \frac{UA_e}{2} \{ (T_{he_2} - T_{ce_2}) + (T_{he_1} - T_{ce_1}) \} \dots\dots (4.52)$$

Eqs (4.49), (4.50) and (4.52) give

$$C_{ce}(T_{ce2} - T_{ce1}) = C_{he}(T_{he1} - T_{he2}) =$$

$$= UA_e \frac{1}{2} \{ (T_{he2} - T_{ce2}) + (T_{he1} - T_{ce1}) \} \dots\dots\dots(4.53)$$

For cold side

$$T_{ce2} - T_{ce1} = \frac{UA_e}{2C_{ce}} \{ (T_{he2} - T_{ce2}) + (T_{he1} - T_{ce1}) \} \dots\dots(4.54)$$

$$T_{ce2} = \frac{UA_e}{2C_{ce}} T_{he2} - \frac{UA_e}{2C_{ce}} T_{ce2} +$$

$$+ \frac{UA_e}{2C_{ce}} T_{he1} - \frac{UA_e}{2C_{ce}} T_{ce1} + T_{ce1} \dots\dots\dots(4.55)$$

$$(1 + \frac{UA_e}{2C_{ce}}) T_{ce2} = \frac{UA_e}{2C_{ce}} T_{he2} + \frac{UA_e}{2C_{ce}} T_{he1} +$$

$$+ (1 - \frac{UA_e}{2C_{ce}}) T_{ce1} \dots\dots\dots(4.56)$$

Subtract $(1 + \frac{UA_e}{2C_{ce}}) T_{cin} = \frac{UA_e}{2C_{ce}} T_{cin} + \frac{UA_e}{2C_{ce}} T_{cin} +$

$$+ (1 - \frac{UA_e}{2C_{ce}}) T_{cin} \text{ and divide by } T_{hin} - T_{cin}$$

$$(1 + \frac{UA_e}{2C_{ce}}) \frac{T_{ce2} - T_{cin}}{T_{hin} - T_{cin}} = \frac{UA_e}{2C_{ce}} \frac{T_{he2} - T_{cin}}{T_{hin} - T_{cin}} +$$

$$+ \frac{UA_e}{2C_{ce}} \frac{T_{he1} - T_{cin}}{T_{hin} - T_{cin}} + (1 - \frac{UA_e}{2C_{ce}}) \frac{T_{ce1} - T_{cin}}{T_{hin} - T_{cin}} \dots\dots\dots(4.57)$$

Eqs (4.57) and (4.14) give

$$\phi_{c_2} = a_3 \phi_{h_2} + b_3 \phi_{h_1} + c_3 \phi_{c_1} \dots\dots\dots (4.58)$$

, where

$$a_3 = \frac{\frac{UA_e}{2C_{ce}}}{1 + \frac{UA_e}{2C_{ce}}} \dots\dots\dots (4.59)$$

$$b_3 = \frac{\frac{UA_e}{2C_{ce}}}{1 + \frac{UA_e}{2C_{ce}}} = a_3 \dots\dots\dots (4.60)$$

$$c_3 = \frac{1 - \frac{UA_e}{2C_{ce}}}{1 + \frac{UA_e}{2C_{ce}}} \dots\dots\dots (4.61)$$

For hot side Eq (4.53) gives

$$C_{he}(T_{he_1} - T_{he_2}) = UA_e \frac{1}{2} \{ (T_{he_2} - T_{ce_2}) + (T_{he_1} - T_{ce_1}) \} \dots\dots\dots (4.62)$$

$$T_{he_1} - T_{he_2} = \frac{UA_e}{2C_{he}} \{ (T_{he_2} - T_{ce_2}) + (T_{he_1} - T_{ce_1}) \} \dots\dots (4.63)$$

$$\begin{aligned} \left(1 + \frac{UA_e}{2C_{he}}\right) T_{he_2} &= \frac{UA_e}{2C_{he}} T_{ce_2} + \frac{UA_e}{2C_{he}} T_{ce_1} + \\ &+ \left(1 - \frac{UA_e}{2C_{he}}\right) T_{he_1} \dots\dots\dots (4.64) \end{aligned}$$

$$\text{Subtract } \left(1 + \frac{UA_e}{2C_{he}}\right) T_{cin} = \frac{UA_e}{2C_{he}} T_{cin} + \frac{UA_e}{2C_{he}} T_{cin} +$$

$$+ \left(1 - \frac{UA_e}{2C_{he}}\right) T_{cin} \text{ and divide by } T_{hin} - T_{cin}$$

$$\left(1 + \frac{UA_e}{2C_{he}}\right) \frac{T_{he2} - T_{cin}}{T_{hin} - T_{cin}} = \frac{UA_e}{2C_{he}} \frac{T_{ce2} - T_{cin}}{T_{hin} - T_{cin}} +$$

$$+ \frac{UA_e}{2C_{he}} \frac{T_{ce1} - T_{cin}}{T_{hin} - T_{cin}} + \left(1 - \frac{UA_e}{2C_{he}}\right) \frac{T_{he1} - T_{cin}}{T_{hin} - T_{cin}} \dots\dots\dots (4.65)$$

Eqs (4.65) and (4.14) give

$$\phi_{h2} = e_3 \phi_{c2} + f_3 \phi_{c1} + g_3 \phi_{h1} \dots\dots\dots (4.66)$$

, where

$$e_3 = \frac{\frac{UA_e}{2C_{ce}} \cdot \frac{C_{ce}}{C_{he}}}{1 + \frac{UA_e}{2C_{ce}} \cdot \frac{C_{ce}}{C_{he}}} \dots\dots\dots (4.67)$$

$$f_3 = \frac{\frac{UA_e}{2C_{ce}} \cdot \frac{C_{ce}}{C_{he}}}{1 + \frac{UA_e}{2C_{ce}} \cdot \frac{C_{ce}}{C_{he}}} \dots\dots\dots (4.68)$$

$$g_3 = \frac{1 - \frac{UA_e}{2C_{ce}} \cdot \frac{C_{ce}}{C_{he}}}{1 + \frac{UA_e}{2C_{ce}} \cdot \frac{C_{ce}}{C_{he}}} \dots\dots\dots (4.69)$$

4.4.4 Method 4

Method 4 is based on the arithmetic mean of ϕ from method one (I) and two (II). For method one Eqs (4.22) and (4.28) give

$$\phi_{c2I} = a_1 \phi_{c1} + b_1 \phi_{h2} \dots\dots\dots (4.70)$$

$$\phi_{h2I} = e_1 \phi_{h1} + f_1 \phi_{c2} \dots\dots\dots (4.71)$$

For method 2 Eqs (4.39) and (4.46) give

$$\phi_{c2II} = a_2 \phi_{h1} + b_2 \phi_{c1} \dots\dots\dots (4.72)$$

$$\phi_{h2II} = e_2 \phi_{h1} + f_2 \phi_{c1} \dots\dots\dots (4.73)$$

The arithmetic mean of Eqs (4.70) and (4.72) is

$$\begin{aligned} \phi_{c2} &= \frac{1}{2} (\phi_{c2I} + \phi_{c2II}) = \frac{1}{2} (a_1 + b_2) \phi_{c1} + \\ &+ \frac{1}{2} a_2 \phi_{h1} + \frac{1}{2} b_1 \phi_{h2} \dots\dots\dots (4.74) \end{aligned}$$

The arithmetic mean of Eqs (4.71) and (4.73) is

$$\begin{aligned} \phi_{h2} &= \frac{1}{2} (\phi_{h2I} + \phi_{h2II}) = \frac{1}{2} (e_1 + e_2) \phi_{h1} + \\ &+ \frac{1}{2} f_2 \phi_{c1} + \frac{1}{2} f_1 \phi_{c2} \dots\dots\dots (4.75) \end{aligned}$$

4.4.5 Methods 5 and 6

Methods 5 and 6 are the Mason [4] formula used in two ways for an element. Eqs (3.13) and (3.12) give

$$\phi_{c2} = \phi_{c1} + \eta_{ce} (\phi_{h1} - \phi_{c1}) \dots\dots\dots (4.76)$$

$$\phi_{h2} = \phi_{h1} - \eta_{he} (\phi_{h1} - \phi_{c1}) \dots\dots\dots (4.77)$$

η_{ce} and η_{he} are calculated as in section 3.1, Eqs (3.1) through (3.6), where for the special case $\frac{C_c}{C_h} = 0$ is [12, p 18] the solution for all heat exchangers

$$\epsilon = 1 - e^{-NTU} \dots\dots\dots (4.78)$$

In method 5 real variables are used and in method 6 only integers are used. Thus method 6 is the same method as the one in section (3.3) through (3.5).

4.4.6 Method 7

Method 7 is just the formula by Mason, Eqs (3.1) through (3.6), applied to the whole heat exchanger. This method is just a reference calculation, so that it will be possible to see the magnitude of the error in the calculations by methods 1 to 6.

4.4.7 The usefulness of methods 1 to 6

If the only aim is to calculate the effectiveness of a crossflow heat exchanger for single phase flow, method 7 can be used. If, however, temperature profiles are of importance, for example if there is a change in phase, we have to use one of the methods 1 to 6. By methods 1 to 6 the line for change in phase can be located and after that a different way of calculation can be used, not necessarily the NTU-method. Another case when method 7 cannot be used is when the inlet mass velocity profiles or temperature profiles are not uniform.

4.5 Calculation of the whole heat exchanger

The results for the whole heat exchanger of methods 1 to 6 will now be compared with the result of method 7.

From Fig 13 it can be seen that the heat exchanger has been divided into $N \times N$ elements. But this is not necessary, the heat exchanger can equally well be divided into $N \times M$ elements. In methods 1 to 6 the different elements can

be connected by means of their outlet and inlet temperatures (See chapter 3) and from that, temperature profiles and later the effectiveness can be calculated. In this chapter the temperatures at the inlet are assumed to be uniform, so that methods 1 to 6 can be directly compared with method 7, but this is not necessary (See chapter 3). The inlet temperature condition on the cold side is $\phi_{c_{in}}$ and on the hot side $\phi_{h_{in}}$, where

$$\left. \begin{aligned} \phi_{c_{in}} &= \frac{T_{c_{in}} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} = 0 \\ \phi_{h_{in}} &= \frac{T_{h_{in}} - T_{c_{in}}}{T_{h_{in}} - T_{c_{in}}} = 1 \end{aligned} \right\} \dots\dots\dots (4.79)$$

Method 1

Eqs (4.22) and (4.28) give

$$\phi_{c_{i,j}} = a_1 \phi_{c_{i,j-1}} + b_1 \phi_{h_{i,j}} \dots\dots\dots (4.80)$$

$$\phi_{h_{i,j}} = e_1 \phi_{h_{i-1,j}} + f_1 \phi_{c_{i,j}} \dots\dots\dots (4.81)$$

Method 2

Eqs (4.39) and (4.46) give

$$\phi_{c_{i,j}} = a_2 \phi_{h_{i-1,j}} + b_2 \phi_{c_{i,j-1}} \dots\dots\dots (4.82)$$

$$\phi_{h_{i,j}} = e_2 \phi_{h_{i-1,j}} + f_2 \phi_{c_{i,j-1}} \dots\dots\dots (4.83)$$

Method 3

Eqs (4.53) and (4.66) give

$$\phi_{c_{i,j}} = a_3 \phi_{h_{i,j}} + b_3 \phi_{h_{i-1,j}} + c_3 \phi_{c_{i,j-1}} \dots\dots\dots (4.84)$$

$$\phi_{h_{i,j}} = e_3 \phi_{c_{i,j}} + f_3 \phi_{c_{i,j-1}} + g_3 \phi_{h_{i-1,j}} \dots\dots\dots (4.85)$$

Method 4

Eqs (4.74) and (4.75) give

$$\phi_{c_{i,j}} = \frac{1}{2}(a_1 + b_2)\phi_{c_{i,j-1}} + \frac{1}{2} a_2 \phi_{h_{i-1,j}} + \frac{1}{2} b_1 \phi_{h_{i,j}} \quad (4.86)$$

$$\phi_{h_{i,j}} = \frac{1}{2}(e_1 + e_2)\phi_{h_{i-1,j}} + \frac{1}{2} f_2 \phi_{c_{i,j-1}} + \frac{1}{2} f_1 \phi_{c_{i,j}} \quad (4.87)$$

Methods 5 and 6

See section 4.4.5.

Method 7

Mason's [4] method for the whole heat exchanger.

See section 4.4.6 and Appendix 1, column 7 in [15] and also Fig 35.

4.6 Preliminary results

4.6.1 Limits of calculations

The results for the whole heat exchanger from the first six methods are compared with results from method 7 in order to get a first classification of the methods and their usefulness. The number of elements N in one direction was given the value of 5, 10, 20, 30, 40 and 50. The total NTU-value [Eq (3.2)] was changed from 0.5 to 10.0 in intervals of 0.5. The capacity ratio C_c/C_h was changed from 0.0 to 1.0 in intervals of 0.1; thus $C_c = C_{\min}$. The first aim of the calculation was to explore the different methods, and find out how the number of elements in the heat exchanger influenced the accuracy of the calculation. The effectiveness $\epsilon = \eta_c$ [Eqs (3.7) and (3.6)] of the heat exchanger was calculated in the following way [Eqs (3.6) and (4.14), Fig 13]

$$\epsilon = \eta_c = \frac{1}{N} \sum_{i=1}^N \phi_{c_{i,N}} \quad \dots\dots\dots (4.88)$$

This is possible because the inlet mass velocities and temperatures are uniform so that $\phi_{c_{in}} = 0$ and $\phi_{h_{in}} = 1$ (Eq (4.79)). The intention was that the calculations should

not differ from the result by Mason by more than 1 %. Another intention was that the methods should not be unstable in any way.

4.6.2 Method 1

Method 1 gave, as was expected, an effectiveness which was low compared with that of Mason's method, because the average mean temperature difference was chosen as the difference between the hot temperature and the cold temperature at the outlet of the element. The results are given in [15, Appendix 1, column marked 1]. They differ from the Mason value by more than 1.0 %, even when the number of elements in one direction is as high as 50. The method is not considered good enough. The method can however be used for approximate calculations and it is stable in the NTU-interval considered, 0.5-10.0.

4.6.3 Method 2

It can be seen from [15, Appendix 1, column 2] that method 2 sometimes gives $\epsilon > 1$, which is physically impossible. This is probably connected with the fact that b_2 and e_2 (Eqs (4.41) and (4.47)) can become negative. Thus method 2 should not be used.

4.6.4 Method 3

The accuracy of method 3 can be so high that the error in the effectiveness is in the fifth figure for total NTU-values of 10 and C_c/C_h -values of 1.0. This occurs when the number of elements in one direction is 50. But this method is also accurate for fewer elements. If the number of elements in one direction is five, the error is less than or equal to 0.6 %. See [15, Appendix 1, column 3]. Method 3 is thus excellent and will be compared with other methods later in section 4.7.

4.6.5 Method 4

It can be seen from [15, Appendix 1, column 4] that method 4 sometimes gives $\epsilon > 1$, which is physically impossible. Thus method 4 should not be used.

4.6.6 Method 5

The use of the formula given by Mason in an element calculation gives a good result and when real variables are used it is one of the best methods. The use of method 5 will be discussed later in section 4.7. For results, see [15, Appendix 1, column 5].

4.6.7 Method 6

Method 6 is the same as method 5 except that it is based on an integer calculation with values between 0 and 10^4 . The method is faster than method 5, but it has a somewhat larger error. It can be used if faster calculations are wanted. See [15, Appendix 1, column 6].

4.6.8 Iterative and direct calculations

Methods 1, 3 and 4 are iterative, which means that the temperatures in each element first have to be assumed e.g. all $T_{ce} = 0$ and $T_{he} = 1$. Method 2 is direct. Methods 5 and 6 are also direct, but when calculating the effectiveness a sufficiently large number of terms has to be included so as to make the error sufficiently small [4, and section 3.1]. In the first calculations the errors in the effectiveness of all six methods have been about 10^{-5} for each element.

From these preliminary calculations the best methods, that is methods 3 and 5, were selected and these methods were used in a final test.

4.7

Final results

4.7.1 Methods 3 and 5

In the final test the accuracy was chosen to be 10^{-30} in each calculation. It must be remembered, however, that the computer just has approximately seven significant figures. The calculations are given in [15, Appendix 2]. The input values are the same as in the preliminary calculations (Section 4.6.1).

4.7.2 Results

Methods 5 and 3 are both very accurate when they are compared with method 7 (Mason). In Table 4.7.1 the results for total NTU = 10.0, $C_c/C_h = 0.0$ to 1.0 are given for 5, 10, 20, 30, 40 and 50 elements in one direction.

Table 4.7.1 Effectiveness ϵ of crossflow heat exchangers

No. of elements	NTU	C_c/C_h	Method 3	Method 5	Method 7
5	10.0	0.00	1.000000	0.999955	0.999955
5	10.0	0.10	0.999960	0.998961	0.999261
5	10.0	0.20	0.999159	0.995551	0.996853
5	10.0	0.30	0.995701	0.988258	0.991430
5	10.0	0.40	0.987575	0.976008	0.981792
5	10.0	0.50	0.973556	0.958311	0.967096
5	10.0	0.60	0.953390	0.935281	0.947018
5	10.0	0.70	0.927601	0.907513	0.921762
5	10.0	0.80	0.897180	0.875928	0.891972
5	10.0	0.90	0.863318	0.841591	0.858593
5	10.0	1.00	0.827211	0.805579	0.822714
10	10.0	0.00	0.999983	0.999955	0.999955
10	10.0	0.10	0.999509	0.999177	0.999261
10	10.0	0.20	0.997495	0.996454	0.996853
10	10.0	0.30	0.992510	0.990383	0.991430
10	10.0	0.40	0.983202	0.979754	0.981792
10	10.0	0.50	0.968658	0.963827	0.967096
10	10.0	0.60	0.948560	0.942446	0.947018
10	10.0	0.70	0.923179	0.915997	0.921762
10	10.0	0.80	0.893237	0.885274	0.891972
10	10.0	0.90	0.859740	0.851312	0.858593
10	10.0	1.00	0.823804	0.815226	0.822714
20	10.0	0.00	0.999964	0.999954	0.999955
20	10.0	0.10	0.999327	0.999238	0.999261
20	10.0	0.20	0.997016	0.996740	0.996853
20	10.0	0.30	0.991700	0.991127	0.991430
20	10.0	0.40	0.982142	0.981184	0.981792
20	10.0	0.50	0.967484	0.966094	0.967096
20	10.0	0.60	0.947402	0.945582	0.947018
20	10.0	0.70	0.922114	0.919910	0.921762
20	10.0	0.80	0.892286	0.889779	0.891972
20	10.0	0.90	0.858878	0.856170	0.858593
20	10.0	1.00	0.822985	0.820186	0.822714

Table 4.7.1 Effectiveness ϵ of crossflow heat exchangers

(continued)

No. of elements	NTU	C_c/C_h	Method 3	Method 5	Method 7
30	10.0	0.00	0.999959	0.999954	0.999955
30	10.0	0.10	0.999290	0.999250	0.999261
30	10.0	0.20	0.996925	0.996800	0.996853
30	10.0	0.30	0.991551	0.991288	0.991430
30	10.0	0.40	0.981946	0.981505	0.981792
30	10.0	0.50	0.967268	0.966619	0.967096
30	10.0	0.60	0.947188	0.946330	0.947018
30	10.0	0.70	0.921919	0.920868	0.921762
30	10.0	0.80	0.892112	0.890907	0.891972
30	10.0	0.90	0.858721	0.857410	0.858593
30	10.0	1.00	0.822835	0.821473	0.822714
40	10.0	0.00	0.999957	0.999954	0.999955
40	10.0	0.10	0.999275	0.999254	0.999261
40	10.0	0.20	0.996893	0.996822	0.996853
40	10.0	0.30	0.991497	0.991348	0.991430
40	10.0	0.40	0.981880	0.981625	0.981792
40	10.0	0.50	0.967193	0.966819	0.967096
40	10.0	0.60	0.947113	0.946616	0.947018
40	10.0	0.70	0.921850	0.921238	0.921762
40	10.0	0.80	0.892050	0.891346	0.891972
40	10.0	0.90	0.858666	0.857896	0.858593
40	10.0	1.00	0.822782	0.821980	0.822714
50	10.0	0.00	0.999955	0.999954	0.999955
50	10.0	0.10	0.999269	0.999256	0.999261
50	10.0	0.20	0.996877	0.996832	0.996853
50	10.0	0.30	0.991472	0.991377	0.991430
50	10.0	0.40	0.981847	0.981683	0.981792
50	10.0	0.50	0.967158	0.966915	0.967096
50	10.0	0.60	0.947078	0.946755	0.947018
50	10.0	0.70	0.921815	0.921419	0.921762
50	10.0	0.80	0.892019	0.891561	0.891972
50	10.0	0.90	0.858637	0.858135	0.858593
50	10.0	1.00	0.822755	0.822231	0.822714

For five elements in one direction it can be seen that the accuracy of methods 3 and 5 is already good. Method 3 gives an error of approximately 0.6 % in the worst case and method 5 1.7 %. For ten elements the values are 0.15 and 0.6 % respectively. With 50 elements the results are 0.004 and 0.05 % respectively. The most interesting thing about this result is not only the fact that we can use as few as five elements in one direction and still get a result which is considered useful (less than 1 % error), but also the fact that by

using the approximation that the mean temperature difference is equal to the arithmetic mean of the inlet difference and outlet difference between the hot and cold temperatures (method 3), we can get a result which is better than the results obtained by using the Mason formula for each element (method 5). Another interesting result is that the effectiveness according to method 3 is always slightly higher and according to method 5 always slightly lower than the effectiveness given by Mason (method 7).

4.7.3 Conclusions

This chapter had as an aim to find a satisfactory way of calculating the effectiveness of a crossflow heat exchanger without mixing. Six different methods were discussed. It was found that method 3 was the best one to use. Method 3 will not be used when the inlet mass flow and temperature profiles are uniform, because then Mason's formula for the whole heat exchanger is better. Method 3 will, however, be used for non-uniform inlet profiles, in which case it is better than method 5. Method 3 will also be used when the temperature profiles of the heat exchanger are wanted. Method 3 can therefore be very important if two phase flow is obtained in the heat exchanger, when thus the Mason formula for the whole heat exchanger cannot be used. In section 6.2.2 the results will be compared with those of Sahlberg [8]. In the case of non-uniform inlet temperatures Eq (4.14) can be replaced by the following way

$$\left. \begin{aligned} \phi_{c i, 0} &= \frac{T_{c e i, 0} - T_{c i n, m i n}}{T_{h i n, m a x} - T_{c i n, m i n}} \\ \phi_{h 0, j} &= \frac{T_{h e 0, j} - T_{c i n, m i n}}{T_{h i n, m a x} - T_{c i n, m i n}} \end{aligned} \right\} \dots\dots\dots (4.89)$$

5. MULTIPASS COUNTERCURRENT ARRANGEMENTS

This chapter presents a method of calculating temperature profiles in multipass crossflow heat exchangers, which are working in a countercurrent arrangement. The method is iterative. In this chapter the assumption is made that no mixing takes place between the passes. The method is a rather powerful tool to calculate temperature profiles in this kind of arrangements. The method can also be used when mixing occurs between passes. This chapter is a direct continuation of chapter 3.

5.1 Type of heat exchangers

Both the inlet mass velocity and temperature profiles can vary. The calculations will be made in the same way as in section 3.2 by a step-by-step method in each heat exchanger. But as the heat exchangers are in a countercurrent arrangement, the whole arrangement has to be calculated in an iterative way. In this chapter up to four crossflow heat exchangers in countercurrent arrangement will be discussed, but the method can be used for a larger number and also mixing between the passes can be included.

5.2 Method of calculation

Single pass

The temperature profiles in a crossflow heat exchanger can be calculated in the following way from Eqs (3.23) and (3.24)

$$T_{c_{i,j}} = T_{c_{i,0}} + \sum_{j=1}^j \eta_{c_{i,j}} (T_{h_{i-1,j}} - T_{c_{i,j-1}}) \dots\dots (5.1)$$

$$T_{h_{i,j}} = T_{h_{0,j}} - \sum_{i=1}^i \eta_{h_{i,j}} (T_{h_{i-1,j}} - T_{c_{i,j-1}}) \dots\dots (5.2)$$

Eqs (5.1) and (5.2) can be solved if the element inlet conditions of temperatures and mass velocities are known, so that the NTU-values of both the hot and the cold sides can be calculated.

5.3 Multipass arrangements

The temperature profiles in different arrangements of crossflow heat exchangers have been treated by Sahlberg [8] in 1957. His solution is mathematically clear but is more difficult to use than the present method.

5.4 General inlet data

Eqs (3.16) and (3.17) give the following: The temperature and capacity profiles at the inlet are represented by vectors (chapter 3).

$$C_h = (C_{he}(1), C_{he}(2), \dots, C_{he}(N))$$

$$C_c = (C_{ce}(1), C_{ce}(2), \dots, C_{ce}(N))$$

$$T_h = (T_{h0,1}, T_{h0,2}, \dots, T_{h0,N})$$

$$T_c = (T_{c1,0}, T_{c2,0}, \dots, T_{cN,0})$$

Each crossflow heat exchanger is divided into $N \times N$ elements. If the expression $A_{i,j} \cdot U$ is known, the effectiveness and the temperature profiles can be calculated.

5.5 Two pass arrangement

Fig 14 shows the locations of the inlet profiles in a two pass arrangement. Pass 1 is the first pass that the cold profiles enter and pass 2 is the first pass the hot profiles enter. The first problem is which temperature is obtained at the location a in Fig 14, if no mixing takes place between passes. This problem can be solved if a certain hot temperature profile is assumed to enter pass 1.

For example this temperature profile can be assumed to be the same as the one which enters pass 2. The same mass velocity profile enters pass 1 as pass 2 except that it has to be rotated so that subscripts are interchanged in Fig 14 if no mixing occurs. The inlet conditions for pass 1 are now known and if $\frac{A_{i,j} \cdot U}{C_p}$ is known the calculation can begin.

The way to calculate the temperature profiles is described in section 3.2. Thus from such a calculation the outlet temperatures from pass 1 on the cold and hot sides are equal to the temperatures from the element (1,N) to (N,N) on the cold side and the element (N,1) to (N,N) on the hot side respectively. The outlet temperatures on the cold side and the corresponding mass velocities are, after rotation of the two vectors, the inlet conditions of pass 2 (see Fig 14). The outlet temperatures on the hot side are the first approximation of the hot temperatures from the whole arrangement. Pass 2 can now be calculated, when all inlet conditions are known. The inlet conditions on the hot side are the given boundary conditions. For the cold side of pass 2 the calculation of pass 1 gives the inlet conditions. When pass 2 is calculated, the outlet temperatures are obtained in exactly the same way as for pass 1.

The outlet temperature on the cold side from pass 2 is the first approximation of the temperature on the cold side from the arrangement. The hot outlet temperature and mass velocity profiles have to be rotated, and after that they represent the second approximation of the inlet conditions on the hot side for pass 1. The calculations can now go on with new approximations for pass 1 and 2, until the error between two iterations has become sufficiently small.

Fig 15 and 16 show the locations of the inlet profiles for a three and four pass arrangement. The calculations are made in the same way as for two passes. The only difference is that assumptions have to be made about the inlet conditions of the hot side for the third and fourth passes, but these assumptions can be made in the same way as was done above for the second pass in a two pass arrangement. For example the assumed inlet temperature of the hot side of pass 3 can be the same as the inlet temperature of the hot side of pass 4. After the first calculations have been made, iterations can continue until the error is sufficiently small between two calculations.

5.7

Results

In this chapter the main object has been to find a fast way of calculating temperatures, and not at this stage to get the best accuracy. A computer program with integer temperatures between 10000 and 0 was used. The iteration procedure was stopped when the difference between two iterations was 0.1 % of the maximum value 10 000 (hot inlet temperature). This probably means that the error in the temperature is of the same magnitude as that in section 3.5, Table 3.5.1, i.e. about 0.3 %. In Fig 17 through 24 the number of elements for each pass in the heat exchangers has been 20×20 , the total $NTU_h = 1.0$ for the hot side, and $NTU_c = 1.0$ for the cold side. This means that for each element's $NTU_{he} = NTU_{ce} = \frac{1.0}{20} = 0.05$ according to Eq (4.4). The inlet temperatures were 10 000 on the hot side and 0 on the cold side in all cases. The layout of the figures has been chosen in such a way so that it would be easy to compare them with Fig 14 through 16. The flow directions are the same in Fig 17 through 24 as in Fig 14 through 16.

Fig 17 and 18 give the result from the calculation of the two pass heat exchanger. Fig 17 is the cold side and Fig 18 gives the hot side. The temperature profiles for the three pass heat exchanger are given in Fig 19 through 21, where Fig 19 gives the cold temperature and Fig 20 through 21 give the hot temperature. Fig 22 through 24 show the temperature profiles of a four pass heat exchanger. Fig 22 gives the cold side and Fig 23 and 24 give the hot side.

Fig 25 through 27 show a rather complicated case of a four pass heat exchanger, where neither the temperature profiles nor the mass velocity profiles are uniform. The following inlet temperature profiles are chosen.

$$T_{c_{i,0}} = 0.03 \cdot i \cdot 10\ 000$$

$$T_{h_{0,j}} = (0.8 + 0.01 \cdot j) \cdot 10\ 000$$

, where i and j are integers refering to the element number (i,j) .

The mass flow rate profiles are chosen to be:

$$\dot{m}_{ce_{i,0}} = (0.8 + 0.01 \cdot i)$$

$$\dot{m}_{he_{0,j}} = (0.5 + 0.025 \cdot j)$$

$$\frac{A_{i,j} \cdot U}{C_p} = 0.1$$

The calculation of the NTU-numbers is performed in the same way as in section 3.5. The profiles are chosen arbitrarily in order to show that the method also works for complex patterns.

Fig 28 shows an isotherm in the heat exchanger located at the temperature level 5 500 (highest value 10 000). This sort of picture can be used if one is interested in the behaviour of the fluid over or under a certain temperature level.

From above it can be seen that the method is powerful, when temperature profiles have to be calculated. The method can also be used when mixing takes place. In this report each step has been divided into 20×20 elements but the number of elements can be increased if a more accurate solution is wanted.

6. SUMMARY OF RESULTS FROM CALCULATIONS WITHOUT LONGITUDINAL HEAT CONDUCTION

In this chapter a summary of the previous results regarding the element method is given.

6.1 Calculation schemes

In the previous chapters 3, 4 and 5 different ways of calculating the effectiveness and temperature profiles in a crossflow heat exchanger were presented. The result was that the two best ways of calculating the effectiveness and temperature profiles of a single pass crossflow heat exchanger without mixing and with uniform inlet profiles are the methods 3 and 5 of chapter 4. The methods can also be used together with an iteration process (section 5.5 and 5.6) for multipass countercurrent arrangements, with different assumptions about the mixing between passes. The results will be compared with those of Sahlberg [8].

6.2 Single pass

6.2.1 Effectiveness

In chapters 3 and 4 [13, 15] calculations on single pass crossflow heat exchangers were performed. From these calculations it was found that the Mason [4] formula (method 7) was fast and accurate to use, if the effectiveness of the whole heat exchanger was to be calculated.

6.2.2 Temperature profiles

It is sometimes of importance to see how the temperature profiles in the heat exchanger will vary with the NTU-value and capacity ratio. Therefore the results of methods 3 and 5 of chapter 4 have been compared with the results of Sahlberg [8] for a heat exchanger divided into 20×20 elements and 40×40 elements. The results are given in Table 6.2.1. The total NTU-values were for the hot side $NTU_h = 8.0$ and for the cold side $NTU_c = 4.0$. When

$NTU_c = 4.0$ the "strip NTU_h " will be 0.2, 0.6, 1.0, ..., 7.8, if the heat exchanger is divided into 20×20 elements (Table 3.5.1 in section 3.5). If the heat exchanger is divided into 40×40 elements the "strip NTU_h " will be 0.1, 0.3, 0.5, ..., 7.9.

In order to obtain a direct comparison with the results given by Sahlberg [8] the following formula has been used for the case of 40×40 elements for the dimensionless temperature (Eq (4.14))

$$\phi_{c_{NTU_h}} = \frac{1}{2} (\phi_{c_{NTU_h+0.1}} + \phi_{c_{NTU_h-0.1}}) \dots\dots\dots (6.1)$$

Table 6.2.1 Cold side dimensionless temperatures ϕ .

$$NTU_c = 4.0$$

NTU _h	Sahlberg ⁺	Method 6 ⁺⁺		Method 5		Method 3	
		20×20		Number of		Number of	
		elements	20×20	40×40	20×20	40×40	40×40
0.2	0.9656	0.9628	0.9631	0.9646	0.9646	0.9650	
0.6	0.9255	0.9225	0.9229	0.9246	0.9250	0.9251	
1.0	0.8766	0.8736	0.8740	0.8757	0.8765	0.8764	
1.4	0.8211	0.8182	0.8186	0.8203	0.8213	0.8210	
1.8	0.7610	0.7584	0.7589	0.7604	0.7616	0.7611	
2.2	0.6985	0.6963	0.6968	0.6980	0.6993	0.6987	
2.6	0.6354	0.6337	0.6341	0.6351	0.6362	0.6356	
3.0	0.5731	0.5717	0.5722	0.5729	0.5739	0.5734	
3.4	0.5129	0.5119	0.5124	0.5128	0.5136	0.5131	
3.8	0.4556	0.4550	0.4555	0.4557	0.4563	0.4559	
4.2		0.4017	0.4022	0.4021	0.4025	0.4022	
4.6		0.3523	0.3529	0.3526	0.3528	0.3526	
5.0		0.3072	0.3077	0.3073	0.3073	0.3072	
5.4		0.2663	0.2669	0.2663	0.2662	0.2662	
5.8		0.2297	0.2302	0.2296	0.2293	0.2294	
6.2		0.1970	0.1976	0.1970	0.1966	0.1967	
6.6		0.1683	0.1689	0.1682	0.1677	0.1679	
7.0		0.1431	0.1436	0.1429	0.1424	0.1426	
7.4		0.1211	0.1217	0.1210	0.1204	0.1206	
7.8		0.1020	0.1026	0.1019	0.1014	0.1016	

⁺from [8]

⁺⁺from Table 3.5.1

From Table 6.2.1 it can be seen that method 3 gives a slightly more accurate result than method 5 and method 6.

In chapter 5 [14] the temperature profiles for multipass countercurrent arrangements of crossflow heat exchangers are given for the cases where no mixing takes place between the passes. The calculation schemes are also given in chapter 5. In the present section mixing between the passes is also considered and the results are compared with those of Sahlberg [8]. For each pass the two methods 3 and 5 of chapter 4 [15] were used and for the connection between passes in a multipass arrangement the method of chapter 5 [14] was used. (See Fig 14 through 16). In Table 6.3.1 the values from the calculations when mixing (both hot and cold fluids are mixed) occurs between the passes are compared with the results of Sahlberg and in Table 6.3.2 the same comparison is made for unmixed cold side and mixed hot side between passes. The capacity ratio has been kept constant, equal to 1 in all the calculations. It then follows (Eqs (4.14), (3.5), and (3.7)) that

$$\phi = \eta_c = \eta_h = \varepsilon \quad \dots\dots\dots (6.2)$$

Table 6.3.1 Dimensionless temperature ϕ . Mixing between passes hot and cold side.

$$C_c = C_h \quad \phi = \varepsilon = \eta_c = \eta_h$$

No. of passes	No. of elements in one direction	NTU of each pass	Method 3	Method 5	Sahlberg
2	10	1.0	0.645378	0.645064	0.6452
2	10	2.0	0.761345	0.760520	0.7610
2	10	3.0	0.810822	0.809439	0.8104
2	10	4.0	0.839283	0.837326	0.8388
2	10	5.0	0.858216	0.855678	0.8577
3	10	1.0	0.731893	0.731624	0.7317
3	10	2.0	0.827145	0.826496	0.8269
3	10	3.0	0.865394	0.864342	0.8651
3	10	4.0	0.886790	0.885332	-
3	10	5.0	0.900789	0.898923	0.9004
4	10	1.0	0.784474	0.784242	0.7843
4	10	2.0	0.864503	0.863971	0.8643
4	10	3.0	0.895530	0.894685	0.8953
4	10	4.0	0.912618	0.911461	-
4	10	5.0	0.923700	0.922226	0.9234
2	20	1.0	0.645237	0.645158	0.6452
2	20	2.0	0.761111	0.760893	0.7610
2	20	3.0	0.810532	0.810160	0.8104
2	20	4.0	0.838954	0.838412	0.8388
2	20	5.0	0.857854	0.857129	0.8577
3	20	1.0	0.731772	0.731705	0.7317
3	20	2.0	0.826962	0.826790	0.8269
3	20	3.0	0.865171	0.864890	0.8651
3	20	4.0	0.886545	0.886141	-
3	20	5.0	0.900523	0.899989	0.9004
4	20	1.0	0.784369	0.784311	0.7843
4	20	2.0	0.864353	0.864212	0.8643
4	20	3.0	0.895349	0.895125	0.8953
4	20	4.0	0.912423	0.912104	-
4	20	5.0	0.923490	0.923068	0.9234
2	40	1.0	0.645198	0.645182	0.6452
2	40	2.0	0.761052	0.760997	0.7610
2	40	3.0	0.810459	0.810365	0.8104
2	40	4.0	0.838875	0.838731	0.8388
2	40	5.0	0.857764	0.857570	0.8577
3	40	1.0	0.731735	0.731725	0.7317
3	40	2.0	0.826914	0.826872	0.8269
3	40	3.0	0.865114	0.865046	0.8651
3	40	4.0	0.886487	0.886379	-
3	40	5.0	0.900456	0.900313	0.9004
4	40	1.0	0.784334	0.784329	0.7843
4	40	2.0	0.864314	0.864279	0.8643
4	40	3.0	0.895302	0.895250	0.8953
4	40	4.0	0.912380	0.912293	-
4	40	5.0	0.923437	0.923324	0.9234

Table 6.3.2 Dimensionless temperature ϕ . Cold fluid unmixed and hot fluid mixed between passes.

$$C_c = C_h \quad \phi = \epsilon = \eta_c = \eta_h$$

No. of passes	No. of elements in one direction	NTU of each pass	Method 3	Method 5	Sahlberg
2	10	1.0	0.642246	0.641941	-
2	10	2.0	0.754032	0.753242	0.7537
2	10	3.0	0.801534	0.800198	0.8011
2	10	4.0	0.829227	0.827307	0.8287
2	10	5.0	0.847930	0.845407	-
3	10	1.0	0.728999	0.728738	-
3	10	2.0	0.820141	0.819527	-
3	10	3.0	0.856407	0.855403	-
3	10	4.0	0.877042	0.875621	-
3	10	5.0	0.890824	0.888970	-
4	10	1.0	0.781703	0.781478	-
4	10	2.0	0.858129	0.857629	-
4	10	3.0	0.887446	0.886642	-
4	10	4.0	0.903896	0.902769	-
4	10	5.0	0.914814	0.913348	-
2	20	1.0	0.642087	0.642010	-
2	20	2.0	0.753763	0.753555	0.7537
2	20	3.0	0.801197	0.800838	0.8011
2	20	4.0	0.828839	0.828307	0.8287
2	20	5.0	0.847496	0.846775	-
3	20	1.0	0.728861	0.728796	-
3	20	2.0	0.819925	0.819763	-
3	20	3.0	0.856142	0.855873	-
3	20	4.0	0.876742	0.876350	-
3	20	5.0	0.890492	0.889962	-
4	20	1.0	0.781582	0.781525	-
4	20	2.0	0.857950	0.857817	-
4	20	3.0	0.887226	0.887013	-
4	20	4.0	0.903652	0.903341	-
4	20	5.0	0.914544	0.914125	-
2	40	1.0	0.642043	0.642028	-
2	40	2.0	0.753695	0.753643	0.7537
2	40	3.0	0.801112	0.801021	0.8011
2	40	4.0	0.828744	0.828603	0.8287
2	40	5.0	0.847388	0.847195	-
3	40	1.0	0.728820	0.728811	-
3	40	2.0	0.819870	0.819830	-
3	40	3.0	0.856075	0.856009	-
3	40	4.0	0.876671	0.876566	-
3	40	5.0	0.890409	0.890267	-
4	40	1.0	0.781542	0.781538	-
4	40	2.0	0.857902	0.857870	-
4	40	3.0	0.887170	0.889120	-
4	40	4.0	0.903596	0.903511	-
4	40	5.0	0.914478	0.914365	-

From Tables 6.3.1 and 6.3.2 it can be seen that method 3 gives a slightly more accurate result than method 5.

In chapter 3, 4, 5 [13, 14, 15] and in sections 6.2 and 6.3 the calculation schemes have been developed and tested. Method 3 of chapter 4 has proved to be both easy and accurate to use. It is better than method 5 of chapter 4. The calculation time (Digital Equipment PDP 11/03 computer) for the calculations made with method 5 was found to be approximately 80-100 % longer than that of method 3.

The results of this chapter show that when the temperature profiles are of importance for uniform inlet conditions, method 3 gives accurate results, when compared to previous methods [8, 1, 2]. If the inlet mass velocity profiles and/or inlet temperature profiles are non-uniform this method 3 is also easy to use.

The calculations in chapter 3 through 6 were based on mathematical models, in which only transverse heat conduction in the wall was considered. Below, the longitudinal heat conduction in the wall will also be taken into account. The calculations will be performed for one to four passes, and the multipass arrangements will be countercurrent.

7. CALCULATIONS WITH LONGITUDINAL HEAT CONDUCTION

7.1

Introduction

In this chapter the results from calculations with a numerical solution for crossflow heat exchangers with longitudinal heat conduction (constant temperature through the wall) are given for single pass and multipass heat exchangers with or without mixing between passes on the two sides. No mixing is assumed to take place within the heat exchanger core. The NTU-value per pass varies between 1 and 10. The capacity ratio C_c/C_h is 0.5 or 1.0. The ratio between the heat transfer coefficient on the cold h_c and hot h_h sides is 0.5, 1.0, 1.5, 2.0. The heat exchanger has been divided into 10×10 elements. The differential equations do not seem to be easy to solve analytically in the case of longitudinal heat conduction.

7.2

Assumptions

The following assumptions are made

1. Steady state.
2. Crossflow without mixing within heat exchanger core.
3. No phase change occurs and no heat is generated in the heat exchanger.
4. Heat transfer coefficient is constant throughout the heat exchanger, and the mass velocity is uniform in the heat exchanger.
5. The temperature is constant through the wall.
6. Thermal conductivity k is constant.

The problem of nonuniform inlet temperature and mass velocity (assumption 4) can be solved with the same technique.

In this chapter $T_h = T$ and $T_c = t$. The wall temperature (constant through the wall) is denoted θ .

7.3 Analysis and method of calculation

In a flat plate crossflow heat exchanger the plate separates hot and cold fluids which flow at right angles, Fig 29. For an element $\delta\Delta x\Delta y$ in the flat plate (wall) a heat balance gives

$$q_1 + q_2 + q_3 - q_4 - q_5 - q_6 = 0 \quad \dots\dots\dots(7.1)$$

The mass flow rate on the hot side of the element $\delta\Delta x\Delta y$

$$\dot{m}_{he} = \frac{\dot{m}_h}{L} \cdot \Delta y \quad \dots\dots\dots(7.2)$$

On the cold side the mass flow rate

$$\dot{m}_{ce} = \frac{\dot{m}_c}{L} \cdot \Delta x \quad \dots\dots\dots(7.3)$$

The heat transfer surface area for convection

$$A_e = \Delta x \cdot \Delta y \quad \dots\dots\dots(7.4)$$

The heat transfer cross sectional area for conduction in the flow direction of the hot fluid (x-direction)

$$S_{he} = \delta \cdot \Delta y \quad \dots\dots\dots(7.5)$$

The heat transfer cross sectional area for conduction in the flow direction of the cold fluid (y-direction)

$$S_{ce} = \delta \cdot \Delta x \quad \dots\dots\dots(7.6)$$

The heat flow rate in the wall due to conduction for the element $\delta\Delta x\Delta y$ is in the x-direction

$$\begin{aligned} q_1 - q_4 &= q_{x - \frac{\Delta x}{2}} - q_{x + \frac{\Delta x}{2}} = \\ &= q_x - \frac{\partial q}{\partial x} \frac{\Delta x}{2} - \left(q_x + \frac{\partial q}{\partial x} \frac{\Delta x}{2} \right) = - \frac{\partial q}{\partial x} \Delta x \quad \dots\dots\dots(7.7) \end{aligned}$$

In the y-direction

$$q_2 - q_5 = q_y - \frac{\Delta y}{2} - q_y + \frac{\Delta y}{2} = - \frac{\partial q}{\partial y} \Delta y \quad \dots\dots\dots (7.8)$$

The heat flow rate due to convection (negative z-direction)

$$q_3 = h_h \cdot A_e \cdot (T - \theta) \quad \dots\dots\dots (7.9)$$

$$q_6 = h_c \cdot A_e \cdot (\theta - t) \quad \dots\dots\dots (7.10)$$

q_3 and q_6 can also be expressed as (Eqs (7.2) and (7.4))

$$\begin{aligned} q_3 &= - \dot{m}_{he} C_{p_h} \cdot \frac{\partial T}{\partial x} \Delta x = - \frac{\dot{m}_h}{\ell} C_{p_h} \frac{\partial T}{\partial x} \Delta x \Delta y = \\ &= - \frac{\dot{m}_h}{\ell} C_{p_h} \frac{\partial T}{\partial x} \cdot A_e \quad \dots\dots\dots (7.11) \end{aligned}$$

By means of Eqs (7.3) and (7.4)

$$q_6 = \dot{m}_{ce} C_{p_c} \cdot \frac{\partial t}{\partial y} \Delta y = \frac{\dot{m}_c}{L} C_{p_c} \cdot \frac{\partial t}{\partial y} \Delta x \Delta y = \frac{\dot{m}_c}{L} C_{p_c} \cdot \frac{\partial t}{\partial y} \cdot A_e \cdot (7.12)$$

Fourier's law of conduction

$$q_i = - k S_i \cdot \frac{\partial T}{\partial x_i} \quad \dots\dots\dots (7.13)$$

gives in the x- and y-direction

$$q_x = - k S_{he} \cdot \frac{\partial \theta}{\partial x} \quad \dots\dots\dots (7.14)$$

$$q_y = - k S_{ce} \frac{\partial \theta}{\partial y} \quad \dots\dots\dots (7.15)$$

Eqs (7.7) and (7.14) give

$$q_1 - q_4 = k S_{he} \frac{\partial^2 \theta}{\partial x^2} \Delta x \quad \dots\dots\dots (7.16)$$

Eqs (7.8) and (7.15) give

$$q_2 - q_5 = k S_{ce} \frac{\partial^2 \theta}{\partial y^2} \Delta y \quad \dots\dots\dots (7.17)$$

Eqs (7.1), (7.16), (7.17), (7.11) and (7.12) give

$$kS_{he} \frac{\partial^2 \theta}{\partial x^2} \Delta x +$$

$$+ kS_{ce} \frac{\partial^2 \theta}{\partial y^2} \Delta y - \frac{\dot{m}_h}{\ell} C_{ph} \frac{\partial T}{\partial x} A_e - \frac{\dot{m}_c}{L} C_{pc} \frac{\partial t}{\partial y} A_e = 0 \quad \dots (7.18)$$

Eqs (7.18), (7.4), (7.5) and (7.6) give after division by $\Delta x \Delta y$

$$k \frac{\partial^2 \theta}{\partial x^2} \delta + k \frac{\partial^2 \theta}{\partial y^2} \delta - \frac{\dot{m}_h}{\ell} C_{ph} \frac{\partial T}{\partial x} - \frac{\dot{m}_c}{L} C_{pc} \frac{\partial t}{\partial y} = 0 \quad \dots (7.19)$$

The wall thickness δ can be expressed in two different ways

$$\delta = \frac{S_c}{L} \quad \dots (7.20)$$

$$\delta = \frac{S_h}{\ell} \quad \dots (7.21)$$

Replacing the first δ in Eq (7.19) by means of Eq (7.21) and the second δ by means of Eq (7.20) gives

$$\frac{kS_h}{\ell} \frac{\partial^2 \theta}{\partial x^2} + \frac{kS_c}{L} \frac{\partial^2 \theta}{\partial y^2} - \frac{\dot{m}_h C_{ph}}{\ell} \frac{\partial T}{\partial x} - \frac{\dot{m}_c C_{pc}}{L} \frac{\partial t}{\partial y} = 0 \quad \dots (7.22)$$

Eqs (7.9) and (7.11) give

$$h_h \cdot A_e (T - \theta) = - \frac{\dot{m}_h}{\ell} C_{ph} \cdot \frac{\partial T}{\partial x} \cdot A_e \quad \dots (7.23)$$

Eqs (7.10) and (7.12) give

$$h_c \cdot A_e (\theta - t) = \frac{\dot{m}_c}{L} C_{pc} \cdot \frac{\partial t}{\partial y} \cdot A_e \quad \dots (7.24)$$

Replace x and y by the dimensionless coordinates

$$x' = \frac{x}{L} \quad \dots (7.25)$$

$$y' = \frac{y}{\ell} \quad \dots (7.26)$$

$$\left. \begin{aligned} dx &= L dx' \\ dy &= \ell dy' \end{aligned} \right\} \dots\dots\dots (7.27)$$

Thus Eqs (7.23), (7.24) and (7.22) become

$$\frac{h_h \ell L}{\dot{m}_h C_{p_h}} (T - \theta) + \frac{\partial T}{\partial x'} = 0 \quad \dots\dots\dots (7.28)$$

$$\frac{h_c \ell L}{\dot{m}_c C_{p_c}} (t - \theta) + \frac{\partial t}{\partial y'} = 0 \quad \dots\dots\dots (7.29)$$

$$\frac{k S_h}{\ell L^2} \frac{\partial^2 \theta}{\partial x'^2} + \frac{k S_c}{L \ell^2} \frac{\partial^2 \theta}{\partial y'^2} - \frac{\dot{m}_h C_{p_h}}{\ell L} \frac{\partial T}{\partial x'} - \frac{\dot{m}_c C_{p_c}}{\ell L} \frac{\partial t}{\partial y'} = 0 \quad \dots (7.30)$$

Multiply Eq (7.30) by $\frac{\ell L}{\dot{m}_h C_{p_h}}$

$$\begin{aligned} & \frac{k S_h}{\dot{m}_h C_{p_h} \cdot L} \frac{\partial^2 \theta}{\partial x'^2} + \\ & + \frac{k S_c}{\ell \cdot \dot{m}_c C_{p_c}} \cdot \frac{\dot{m}_c C_{p_c}}{\dot{m}_h C_{p_h}} \frac{\partial^2 \theta}{\partial y'^2} - \frac{\partial T}{\partial x'} - \frac{\dot{m}_c C_{p_c}}{\dot{m}_h C_{p_h}} \frac{\partial t}{\partial y'} = 0 \quad \dots\dots (7.31) \end{aligned}$$

In Eqs (7.28), (7.29) and (7.31) the following dimensionless coefficients are defined as

$$hNTU_h = \frac{h_h \ell L}{\dot{m}_h C_{p_h}} = \frac{h_h A}{\dot{m}_h C_{p_h}} \quad \dots\dots\dots (7.32)$$

$$hNTU_c = \frac{h_c \ell L}{\dot{m}_c C_{p_c}} = \frac{h_c A}{\dot{m}_c C_{p_c}} \quad \dots\dots\dots (7.33)$$

$$\frac{C_c}{C_h} = \frac{\dot{m}_c C_{p_c}}{\dot{m}_h C_{p_h}} \dots\dots\dots (7.34)$$

$$K_h = \frac{k S_h}{\dot{m}_h C_{p_h} L} = \frac{k L \delta}{C_h L} \dots\dots\dots (7.35)$$

$$K_c = \frac{k S_c}{\dot{m}_c C_{p_c} \ell} = \frac{k L \delta}{C_c \ell} \dots\dots\dots (7.36)$$

In Eqs (7.28) through (7.36) C_h , C_c , $\dot{m}_h$, $\dot{m}_c$ and A thus all refer to one plate only or half a channel in a multichannel crossflow heat exchanger.

The boundary conditions are

$$\frac{\partial \theta}{\partial x'} = 0 \quad \text{for } x' = 0, x' = 1 \dots\dots\dots (7.37)$$

$$\frac{\partial \theta}{\partial y'} = 0 \quad \text{for } y' = 0, y' = 1 \dots\dots\dots (7.38)$$

, which means that the edges of the wall are insulated.

$$T_h = T = 1 \quad \text{for } x' = 0 \dots\dots\dots (7.39)$$

$$T_c = t = 0 \quad \text{for } y' = 0 \dots\dots\dots (7.40)$$

, which are the inlet temperature conditions for the fluids.

The set of equations (7.28), (7.29) and (7.31) do not seem to be easy to solve analytically.

7.4 Numerical solution by use of finite-difference methods

Using finite differences Eq (7.28) can by means of Eq (7.32) be expressed as

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta x'} +$$

$$+ hNTU_h \cdot \left(\frac{1}{2} (T_{i,j+1} + T_{i,j}) - \theta_{i,j} \right) = 0 \quad \dots\dots\dots (7.41)$$

$$T_{i,j+1} - T_{i,j} +$$

$$hNTU_h \cdot \Delta x' \cdot \left(\frac{1}{2} (T_{i,j+1} + T_{i,j}) - \theta_{i,j} \right) = 0 \quad \dots\dots\dots (7.42)$$

$$\left(1 + \frac{1}{2} hNTU_h \cdot \Delta x' \right) T_{i,j+1} +$$

$$+ \left(\frac{1}{2} hNTU_h \cdot \Delta x' - 1 \right) T_{i,j} - hNTU_h \cdot \Delta x' \cdot \theta_{i,j} = 0 \quad \dots (7.43)$$

$$\theta_{i,j} + \frac{1 - \frac{1}{2} hNTU_h \cdot \Delta x'}{hNTU_h \cdot \Delta x'} T_{i,j} +$$

$$+ \left[- \frac{1 + \frac{1}{2} hNTU_h \cdot \Delta x'}{hNTU_h \cdot \Delta x'} T_{i,j+1} \right] = 0 \quad \dots\dots\dots (7.44)$$

Eqs (7.29) and (7.33) give in the same way

$$\theta_{i,j} + \frac{1 - \frac{1}{2} hNTU_c \cdot \Delta y'}{hNTU_c \cdot \Delta y'} t_{i,j} +$$

$$+ \left[- \frac{1 + \frac{1}{2} hNTU_c \cdot \Delta y'}{hNTU_c \cdot \Delta y'} t_{i+1,j} \right] = 0 \quad \dots\dots\dots (7.45)$$

Eqs (7.31), (7.34), (7.35) and (7.36) give

$$\begin{aligned}
 & K_h \left[\frac{\theta_{i,j+1} + \theta_{i,j-1} - 2\theta_{i,j}}{\Delta x'^2} \right] + \\
 & + K_c \cdot \frac{C_c}{C_h} \left[\frac{\theta_{i+1,j} + \theta_{i-1,j} - 2\theta_{i,j}}{\Delta y'^2} \right] + \\
 & + \left[- \frac{T_{i,j+1} - T_{i,j}}{\Delta x'} - \frac{C_c}{C_h} \frac{t_{i+1,j} - t_{i,j}}{\Delta y'} \right] = 0 \quad \dots\dots(7.46)
 \end{aligned}$$

7.5 Relations between different dimensionless groups

Eq (1.2) gives

$$\frac{1}{U} = \frac{1}{h_c} + \frac{1}{h_h} \quad \dots\dots\dots(7.47)$$

$$U = \frac{1}{\frac{1}{h_c} + \frac{1}{h_h}} \quad \dots\dots\dots(7.48)$$

If $\dot{m}_c C_{p_c} \leq \dot{m}_h C_{p_h}$ ($C_c \leq C_h$) Eqs (3.2) and (7.48) give

$$NTU = \frac{UA}{C_{\min}} = \frac{UA}{C_c} = \frac{A}{\frac{C_c}{h_c} + \frac{C_c}{h_h}} \quad \dots\dots\dots(7.49)$$

$$NTU \left[\frac{C_c}{Ah_c} + \frac{C_c}{Ah_h} \right] = 1 \quad \dots\dots\dots(7.50)$$

Eqs (7.50) and (7.33) give

$$\frac{NTU}{hNTU_c} \left(1 + \frac{h_c}{h_h} \right) = 1 \quad \dots\dots\dots(7.51a)$$

$$hNTU_c = NTU \left(1 + \frac{h_c}{h_h} \right) \quad \dots\dots\dots(7.51b)$$

Multiplication of Eq (7.50) by $\frac{C_h}{C_c}$ gives on the other hand

$$NTU \frac{C_h}{C_c} \left[\frac{C_c}{Ah_c} + \frac{C_c}{Ah_h} \right] = \frac{C_h}{C_c} \dots\dots\dots (7.52)$$

$$NTU \left[\frac{C_h}{Ah_c} + \frac{C_h}{Ah_h} \right] = \frac{C_h}{C_c} \dots\dots\dots (7.53)$$

Eqs (7.53) and (7.32) give

$$\frac{NTU}{hNTU_h} \left(\frac{h_h}{h_c} + 1 \right) = \frac{C_h}{C_c} \dots\dots\dots (7.54a)$$

$$hNTU_h = NTU \left(\frac{h_h}{h_c} + 1 \right) \frac{C_c}{C_h} \dots\dots\dots (7.54b)$$

Another dimensionless group that can be of importance is (Eqs (7.35), (7.36), (7.49) and (7.32))

$$Y_h = \frac{K_h \cdot C_h}{NTU \cdot C_{\min}} = \frac{kS_h}{LUA} = \frac{k\ell\delta}{LUL\ell} = \frac{k\delta}{UL^2} \dots\dots\dots (7.55)$$

$$Y_c = \frac{K_c \cdot C_c}{NTU \cdot C_{\min}} = \frac{kS_c}{\ell UA} = \frac{kL\delta}{\ell U\ell L} = \frac{k\delta}{UL^2} \dots\dots\dots (7.56)$$

Y_h and Y_c are constant, if U is constant. This case can occur under laminar flow conditions in a heat exchanger with a low value of $\frac{d}{L}$ (section 8.9 and Fig 34). A constant Y -value will therefore represent the "working line" for the heat exchanger in a large part of the laminar regime. Y -values are found in Fig 36-42, 45-61. From Fig 62b it is seen that for the tested heat exchanger Y is practically constant for the test considered in section 8.10. Thus Y is an important characteristic of the crossflow heat exchanger.

The equations (7.28), (7.29) and (7.31) do not involve the overall heat transfer coefficient U , because q_3 is not identical to q_6 (Fig 29). This means that the overall heat transfer coefficient U has to some extent lost its importance, and it can only have a real physical meaning when no longitudinal heat conduction occurs.

7.7

Calculation procedure

For a one pass crossflow heat exchanger Eqs (7.44) through (7.46) can be solved iteratively with the boundary conditions given in Eqs (7.37) through (7.40). The conditions of insulated edges, Eqs (7.37) and (7.38), can be expressed by using four vectors (θ) around the matrix which describes the plate. According to these boundary conditions the values of these vectors equal the values θ of the row or column just inside these vectors in the matrix.

The inlet temperature on the hot side was assumed to be 1 and on the cold side was assumed to be 0 (Eqs (7.39) and (7.40)). These were also the initial temperature values in each element on the hot and cold side respectively.

The initial wall temperature was assumed to be

$$\theta = \frac{h_h}{h_h + h_c} \dots\dots\dots (7.57)$$

This approximative result can be obtained in the following way. If longitudinal heat conduction is neglected

$$h_h(T - \theta) = h_c(\theta - t)$$

Set

$$\begin{cases} T = 1 \\ t = 0 \end{cases}$$

Thus

$$h_h(1 - \theta) = h_c \theta$$

$$h_h = (h_h + h_c)\theta$$

$$\theta = \frac{h_h}{h_h + h_c}$$

Thus the first approximation of T , t , θ is known, and the solution of Eqs (7.44) through (7.46) is obtained in the following way.

Use the old values of θ and T in Eq (7.44), where subscripts are i, j and obtain $T_{i,j+1}$. Do the same thing in Eq (7.45) and obtain $t_{i+1,j}$. In Eq (7.46) use old values for $\theta_{i,j+1}$ and $\theta_{i+1,j}$ and substitute the new values $T_{i,j+1}$ so as to obtain $\theta_{i,j}$. These calculations will be performed for the whole heat exchanger ($i = 1$ to 10 , $j = 1$ to 10).

The calculations are stopped when the difference of the temperatures between two iterations is less than 10^{-5} .

For multipass arrangements the procedure is similar to the one given in chapter 5.

7.8 Calculated arrangement

Calculations have been performed for the following arrangements.

a. One pass.

For two to four passes:

b. No mixing between passes.

c. The fluid on the cold side was mixed between passes.

d. The fluid on the hot side was mixed between passes.

e. Both fluids were mixed between passes.

All calculations for multipass arrangements have been made for the case of countercurrent flow.

7.9.1 Range for K_c and K_h

Some air-to-air heat exchangers will be discussed and a few examples will be given. Air-to-air heat exchangers are often made of aluminum with a wall thickness δ from 0.1 mm to 1.0 mm. The side lengths of the plates are often found in the range of 0.2 m to 1.5 m. The first example chosen is a small heat exchanger with the plate size $L \times l = 0.2 \times 0.2$ m. It is made from $\delta = 0.2$ mm aluminum. The number of plates is 199 and the total air mass flow rate on each side is 0.06 kg/s with a $C_p = 1.005$ kJ/kgK. The thermal conductivity of the aluminum is assumed to be 200 W/mK. The number of channels is thus 198, which means 99 hot channels and 99 cold channels or 198 hot and 198 cold half-channels. Then from Eqs (7.35) and (7.36), which are valid for one plate

$$K_h = K_c = \frac{k \cdot S_h}{\dot{m}_h C_{p_h} \cdot L} = \frac{k \cdot L \cdot \delta}{C_c l} = \frac{k \cdot \delta}{C_c} = \frac{200 \cdot 0.2 \cdot 10^{-3} \cdot 198}{60 \cdot 10^{-3} \cdot 1005} = 0.131$$

If NTU = 2 then from Eqs (7.55) and (7.56)

$$Y_h = Y_c = \frac{K_h \cdot C_h}{NTU \cdot C_{\min}} = \frac{0.131}{2} = 0.066$$

If as a second example this heat exchanger is insted made from plastics with $k = 1$ W/mK and $\delta = 1.0$ mm, then

$$K_h = K_c = \frac{1 \cdot 1 \cdot 10^{-3} \cdot 198}{60 \cdot 10^{-3} \cdot 1005} = 0.00328$$

$$\text{If NTU} = 2 \text{ then } Y_h = Y_c = \frac{0.00328}{2} = 0.0016$$

The third example chosen is an aluminum heat exchanger with plate side $L \times l = 1.0 \times 1.0$ m and thickness $\delta = 0.5$ mm. The number of plates is 221 and on each side passes 3.3 kg/s air. Then

$$K_h = K_c = \frac{200 \cdot 0.5 \cdot 10^{-3} \cdot 220}{3.3 \cdot 1005} = 0.00663$$

$$\text{If NTU} = 2 \text{ then } Y_h = Y_c = \frac{0.00663}{2} = 0.0033$$

These three examples show a common range for air-to-air heat exchangers. For certain applications the mass flow rate can be less, so that K_c and K_h may be larger. Calculations were made for values of K_h from 0.05 to 0.3, with a step of 0.05.

7.9.2 Range of NTU-values

Common air-to-air crossflow heat exchangers have NTU-values from 1.0 to 3.0, but sometimes higher. The range of NTU-values has therefore been chosen between 1 and 10 per pass.

7.9.3 Range of capacity ratio

The capacity ratio $C_{\min}/C_{\max}$ is often about 1.0, but smaller values can also exist. Therefore two values have been chosen, $C_c/C_h = 1.0$ and 0.5.

7.9.4 Heat transfer coefficient ratio

The ratio for the heat transfer coefficient ratio $\frac{h_h}{h_c}$ can vary a great deal. In this report the following values have been used: 0.5 to 2.0 in steps of 0.5.

7.10 Earlier investigations

In the middle of the nineteenseventies J P Chiou, University of Detroit, Detroit, Michigan, made an investigation on how the longitudinal heat conduction affected the effectiveness of single pass crossflow heat exchangers [11]. In the summer of 1978 in Toronto during the Sixth International Heat Transfer Conference these results were discussed by Chiou and the author. Chiou published one way of solving Eqs (7.28) through (7.30) applied to single pass heat exchangers. In this report a more direct method is given and it also includes multipass arrangements. The single pass case has been used to check that the two methods give the same results.

7.11 Results

The results of calculations presented are shown in diagrams in the form:

$$\left. \begin{aligned} \Delta \epsilon_{\text{rel}} &= \frac{\epsilon_{\text{no conduction}} - \epsilon_{\text{with conduction}}}{\epsilon_{\text{no conduction}}} \\ \epsilon_{\text{with conduction}} &= \epsilon_{\text{no conduction}} (1 - \Delta \epsilon_{\text{rel}}) \end{aligned} \right\} \dots (7.58)$$

as a function of different parameters. All results are also given in tables [17].

In [17] and in Fig 35 the reference values for the effectiveness without conduction are given. All values are calculated by means of method 3 of chapter 4. One pass and multipass heat exchangers with mixing between passes are also calculated by means of method 7 of chapter 4 [4]. For multipass arrangement the following expressions are used [12, page 20]

$$\left. \begin{aligned} \epsilon &= \frac{n \cdot \epsilon_p}{1 + (n-1)\epsilon_p} \\ \text{if } C_{\max} &= C_{\min} \end{aligned} \right\} \dots\dots\dots (7.59)$$

and

$$\left. \begin{aligned} \epsilon &= \frac{\left[\frac{1 - \frac{\epsilon_p C_{\min}}{C_{\max}}}{1 - \epsilon_p} \right]^n - 1}{\left[\frac{1 - \frac{\epsilon_p C_{\min}}{C_{\max}}}{1 - \epsilon_p} \right]^n - \frac{C_{\min}}{C_{\max}}} \\ \text{if } C_{\min} &\neq C_{\max} \end{aligned} \right\} \dots\dots\dots (7.60)$$

The reason for doing the calculations for one pass and multipass with mixing between passes both with the difference method 3 and with the method 7 was to see if the accuracy was affected in any way by use of the difference method. The results [17] showed that the loss of accuracy was very small (about 10^{-3} in ϵ) when using the difference method. If a better accuracy has to be achieved, however, the heat exchanger can be divided into a larger number of elements instead of 10×10 , which has been used here.

7.11.1 One pass

The results for one pass show a large decrease in the effectiveness of the heat exchanger. For the case when the capacity on both sides is equal, the decrease is

very significant. As an example a heat exchanger with $K_h = K_c = 0.15$, $h_h = h_c$ and $NTU = 3.0$ will have an effectiveness of approximately 0.682 (Fig 35 and [17] Appendix 1) without conduction. Fig 36 or [17] gives $\Delta\epsilon_{rel} = 0.088$. Thus from Eq 7.58

$$\begin{aligned}\epsilon_{\text{with conduction}} &= (1 - \Delta\epsilon_{rel})\epsilon_{\text{no conduction}} = \\ &= (1 - 0.088)0.682 = 0.622 \quad \dots\dots\dots(7.61)\end{aligned}$$

For the case with $C_c = C_h$ the ratio h_h/h_c between 0.5 and 2.0 has no influence on the effectiveness (Fig 36, 37 and 43). For the case when $C_c/C_h = 0.5$ (Fig 38 through 42 and 44) it is seen that $\Delta\epsilon_{rel}$ is lower than for $C_c/C_h = 1$ for the same h_h/h_c . $\Delta\epsilon_{rel}$ decreases further when h_h/h_c increases. The difference in $\Delta\epsilon_{rel}$ is very small (third digit in $\Delta\epsilon_{rel}$ [17, Appendix 2]) when using method 3 or method 7 of chapter 4 as the reference value $\epsilon_{\text{no conduction}}$.

7.11.2 Asymptotic behavior

From [17, Appendix 2] the conclusion can also be drawn that for the case of $C_c = C_h$ the effectiveness (with conduction) asymptotically goes to a value which is not 1.00 for high NTU-numbers. The asymptotic value depends on the value of K_c and K_h . The higher K_c and K_h are, the lower is the value of the asymptotic effectiveness.

For a heat exchanger which operates at infinite values of NTU, K_c , K_h the longitudinal heat conduction resistance of the wall is zero and at the outlets both the hot and cold fluids have the same temperature as the wall. In other words the heat exchanger can be said to operate as a perfect mixer with θ as the mixing temperature. Thus

$$\theta(C_c + C_h) = C_h \cdot T_{in} + C_c \cdot t_{in} \quad \dots\dots\dots(7.62)$$

$$T_{out} = t_{out} = \theta = \frac{C_h T_{in} + C_c t_{in}}{C_c + C_h} \quad \dots\dots\dots(7.63)$$

If $C_h = C_c$, $T_{in} = 1$ and $t_{in} = 0$ it follows from Eqs (7.63), (4.14), (4.5), (3.6) and (3.7) that

$$\theta = \phi = \eta_c = \eta_h = \varepsilon = 0.5 \quad \dots\dots\dots (7.64)$$

This result can be seen in Fig 62.

7.11.3 Multipass

7.11.3.1 Mixing between passes. The results from the calculations of a single pass crossflow heat exchanger can be used directly for multipass heat exchangers with mixing by use of Eqs (7.59) and (7.60). The results are given in Fig 45 through 59 and in [17, Appendix 2] where the NTU-values mean NTU per pass. The influence of longitudinal heat conduction is smaller, when a multipass arrangement is used. The conclusion regarding the influence of the ratio h_h/h_c is the same as for the single pass heat exchanger. The figures regarding $\Delta\varepsilon_{rel}$ for the case when $C_c/C_h = 0.5$ show a maximum and the value of K_h and K_c determine where the maximum is (Fig 46 through 49, 51 through 54, 56 through 59). In the case $C_c = C_h$ there is no maximum (Fig 45, 50, 55) for $\Delta\varepsilon_{rel}$ in the range of NTU from 1 to 10. It is also obvious that the effectiveness asymptotically goes to a value which is not 1.0 for infinite NTU-values (compare section 7.11.2). If a two pass heat exchanger has a NTU-value per pass equal to 1.2, $C_c = C_h$, $K_h = K_c = 0.15$ and $h_c = h_h$, the effectiveness without conduction (Fig 35 and [17]) is approximately 0.680. Fig 45 then gives $\Delta\varepsilon_{rel} = 0.0285$ and Eq (7.61) gives $\varepsilon_{with\ conduction} = 0.661$. If the influence of conduction between a single pass heat exchanger and a two pass heat exchanger is compared for example from the point of view that the effectiveness without conduction is the same (See results for single pass), it will be found that the influence is much higher for a single pass heat exchanger. This is shown by the example above in section 7.11.1, where $\varepsilon_{no\ conduction} = 0.682$ and $\varepsilon_{with\ conduction} = 0.622$.

7.11.3.2 No mixing, cold side mixed, hot side mixed between passes. For these three cases the same calculations have been made and the results can be found in [17, Appendix 2]. The results are the same as for mixing between passes concerning the shape of the $\Delta\epsilon_{rel}$ -curve. The values differ only slightly from the values for mixing between passes (See Fig 60 and 61).

7.12 Comparison with previously published analyses

The partial differential equation of Nusselt [1] and Mason [4] (no longitudinal heat conduction in the wall) may be obtained by specializing Eqs (7.28) through (7.30) by setting $k = 0$ in these equations. Thus

$$\frac{h_h A}{C_h} (T - \theta) + \frac{\partial T}{\partial x} = 0 \quad \dots\dots\dots (7.65)$$

$$\frac{h_c A}{C_c} (t - \theta) + \frac{\partial t}{\partial y} = 0 \quad \dots\dots\dots (7.66)$$

$$C_h \frac{\partial T}{\partial x} + C_c \frac{\partial t}{\partial y} = 0 \quad \dots\dots\dots (7.67)$$

For the case of no longitudinal heat conduction in the wall

$$q'' = U(T - t) = h_h(T - \theta) = h_c(\theta - t) \quad \dots\dots\dots (7.68)$$

Eqs (7.65), (7.66) and (7.68) give (θ eliminated)

$$UA(T - t) + C_h \frac{\partial T}{\partial x} = 0 \quad \dots\dots\dots (7.69)$$

$$- UA(T - t) + C_c \frac{\partial t}{\partial y} = 0 \quad \dots\dots\dots (7.70)$$

The three equations Eqs (7.67), (7.69) and (7.70) are not independent of each other. Thus additions of Eqs (7.69) and (7.70) give (7.67).

Differentiate Eq (7.69) partially with respect to y' .

Thus

$$\frac{UA}{C_h} \left(\frac{\partial T}{\partial y'} - \frac{\partial t}{\partial y'} \right) + C_h \frac{\partial T}{\partial x' \partial y'} = 0 \quad \dots\dots\dots(7.71)$$

Elimination of $\frac{\partial t}{\partial y'}$ between Eq(7.67) and (7.71) gives

$$\frac{UA}{C_c} \frac{\partial T}{\partial x'} + \frac{UA}{C_h} \frac{\partial T}{\partial y'} + \frac{\partial^2 T}{\partial x' \partial y'} = 0 \quad \dots\dots\dots(7.72)$$

, which corresponds to Eq (8) in Nusselt [1]. The solution of Nusselt [1] is discussed in section 2.1 and 3.1.

Differentiate partially Eq (7.69) with respect to y' and Eq (7.70) with respect to x' , then

$$\frac{UA}{C_h} \frac{\partial}{\partial y'} (T - t) + \frac{\partial^2 T}{\partial x' \partial y'} = 0 \quad \dots\dots\dots(7.73)$$

$$- \frac{UA}{C_c} \frac{\partial}{\partial x'} (T - t) + \frac{\partial^2 t}{\partial x' \partial y'} = 0 \quad \dots\dots\dots(7.74)$$

Subtraction gives

$$\frac{UA}{C_c} \frac{\partial(T - t)}{\partial x'} + \frac{UA}{C_h} \frac{\partial(T - t)}{\partial y'} + \frac{\partial^2(T - t)}{\partial x' \partial y'} = 0 \quad \dots\dots(7.75)$$

, which corresponds to Eq (4) in Mason [4]. The solution of Mason is discussed in section 2.1 and 3.1.

7.13

Summary

The calculations show that the effectiveness of a gas-to-gas crossflow heat exchanger can be strongly dependent upon the longitudinal heat conduction in the wall. The results also show that a multipass countercurrent arrangement reduces the influence of longitudinal heat conduction. This poses the following questions, when high effectiveness is wanted:

- a. Is it possible to reach the effectiveness, which is required, with a single pass heat exchanger?
- b. Does the longitudinal heat conduction mean a very large heat exchanger compared with one where no longitudinal heat conduction has been considered?
- c. Will a multipass heat exchanger use less heat transfer area, because the dependence of longitudinal heat conduction is less?

A test rig, Fig 30, for heat exchangers was used and its different parts are described. The measurements were made automatically (on line) by means of a Digital Equipment PDP 11/05 computer and a Schlumberger data acquisition system. The software was Basic together with routines for the acquisition system. The heat exchanger was supplied with air from two centrifugal fans (Svenska Ziehl EWRE 222-4) and the air heated by two electrical heaters (Vittsjö Elindustrier AB, 1.8 kW and 2.1 kW respectively) controlled by means of adjustable transformers, Fig 30.

8.1

Heat exchanger

The crossflow heat exchanger which was used in the tests, Fig 31, was manufactured just for this investigation. The heat exchanger was made from 0.3 mm SIS 4007-14 aluminium plates 200×200 mm with thermal conductivity $k = 225 \text{ W/mK}$. Brass (assumed $k = 100 \text{ W/mK}$) rods (spacers) 3×3 mm, were used to keep the plates apart. (In commercial crossflow heat exchangers plate corrugations or other kinds of plate deformations are sometimes used instead of spacers). The number of channels in each direction was 28, with four subchannels in each channel. The centerline distance between spacers is then $\frac{197}{4} = 49.25 \text{ mm}$, and the dimensions of each subchannel were thus nominally 46.25×3×200 mm. The spacers were glued to the aluminum plates so that no air should leak from one subchannel to another. The cross-sectional area for the fluid on each side is thus

$$S = 28(0.003 \cdot 0.2 - 5 \cdot 0.003 \cdot 0.003) = 0.01554 \text{ m}^2$$

The heat transfer area is $55 \cdot 0.2 \cdot 0.2 = 2.20 \text{ m}^2$ (insulated outer plates).

The hydraulic diameter

$$d = \frac{4 \cdot S_{\text{subchannel}}}{P_{\text{subchannel}}} = \frac{4 \cdot 3 \cdot 46.25}{2(3 + 46.25)} = 5.63 \text{ mm}$$

The temperatures were measured by 64 thermocouples, 16 on each inlet and outlet side, made from copper-constantan. The reference temperatures for the thermocouples were measured by a platinum, 100 Ω (Pt-100) resistance located in an isothermal box, where the thermocouples were connected to the scanner unit of the data acquisition system. The location of thermocouples can be seen in Fig 30. The thermocouples were calibrated in a thermostatic bath (HETO, Erik Nilsson Instrument AB, Upplands Väsby) and the results were compared with the standard polynomial given in [18]. The standard polynomial did not differ more than ± 0.15 K from the measured values and was therefore used. In the evaluation of tests, the temperature from each thermocouple was chosen to be the mean value of ten separate measurements. The thermocouples were connected in sequential order ten times to avoid any error caused by the influence of temperature change in time. The temperatures in connection with the measurement of the relative humidity were obtained by means of a Pt-100 resistance.

8.3

Mass flow rate measurement

The mass flow through the heat exchanger was measured by means of 32 plastic tubes, 1500 mm long and with a diameter of 16.3 mm. In each tube the static pressure (hole in the wall) and the total pressure (Pitot tube) were measured, Fig 32a. The total pressure was measured in the middle of the tube, 1400 mm ($L/D = 86$) from the inlet, so the velocity profile was fully developed. The dynamic pressure that was obtained in the center of the tube is related to the maximum velocity in the tube by the following equation

$$\frac{p_{\text{dyn}}}{\rho} = \frac{c_{\text{max}}^2}{2} \dots\dots\dots (8.1)$$

For the determination of the average velocity in the measurement tube, an iterative calculation was performed using a set of equations for the ratio of average velocity to maximum velocity as a function of the Reynolds number based on the average velocity (section 8.11). The set of equations was found by regression of the diagrams published by Nikuradse [19], T E Stanton, J R Pannel [20] Fig 32b.

The total pressures from the measurement tubes were connected to a hydraulic multiplexer (Scanivalve Co, San Diego, USA) and the static pressures to another multiplexer. The dynamic pressures (total minus static pressures) from each measurement tube were via a third smaller on-off multiplexer sequentially connected (stepmotors) to a five range micro-manometer (Furness Controls Ltd, Carl-Erik Larsson AB, Solna). The different pressure ranges were obtained by means of another stepmotor. This stepmotor and the on-off multiplexer were operated in such a way that the micro-manometer was not overloaded. The on-off stepmotor operates in such a way that the voltage from the pressure cell of the micromanometer for each dynamic pressure was obtained both at the on and off position. The dynamic pressure reading was then obtained from the difference between these two voltages. This procedure was carefully tested.

The mass flow rate through the heat exchanger could be varied in two different ways. The fan rpm could be controlled by means of an adjustable transformer. Each measurement tube had an adjustable plate at its outlet, Fig 33, so the mass flow rate of each tube could be varied. Thus the prescribed mass velocity profiles could be obtained for the heat exchanger.

8.4 Relative humidity measurement

The relative humidity was obtained by means of two LiCl cells, one in the hot air and one in the cold air (Nova Sina, Switzerland, Carl-Erik Larsson AB, Solna). The

cells were connected to the stepmotor on the micromanometer so this motor was used for more than one purpose. The signal from the two LiCl cells were linearized by means of a transformer. The transformer was connected to the scanner. The best accuracy was achieved on the hot side of the heat exchanger, if the relative humidity was measured before the air was heated because room temperature air (e.g. 20°C, 40 % relative humidity) will have a very low relative humidity when heated to e.g. 50°C for example, and accuracy will then be lower.

The value of the relative humidity given by the two LiCl cells, together with the temperatures where the two cells are located, give the pressure caused by the vapor [21, p 32]. From this pressure the water contents can be calculated and thus the enthalpies of the two air streams (hot and cold) before and after the heat exchanger.

8.5 Range of experimental parameters

The experiments concern the problem of longitudinal heat conduction in the crossflow heat exchanger of Fig 31. The test conditions have been uniform mass velocities and no condensation in the heat exchanger. This was achieved by means of the exit plates of the measurement tubes, at a hot air temperature of approximately 50°C and a cold air temperature of approximately 20°C.

8.6 Mass flow rates

The mass flow rates through the heat exchanger were varied so that the NTU-values were approximately between 0.7 and 12. This means that the mass flow rates in this particular case have to vary from 0.045 kg/s to 0.003 kg/s. In the measurement tubes the dynamic pressure varied between 14 mm Wg (water gauge) to 0.2 mm Wg. The measurements were made in steps of 0.005 kg/s. In all experiments $\dot{m}_h \approx \dot{m}_c$.

The heat exchanger was heavily insulated (10 cm glass fibre wool) so that the loss of heat per unit time from the rig was reduced. This is especially important when measurement with small mass flow rates are made, since then a very limited amount of heat is transferred from the hot side to the cold side per unit time.

The influence of longitudinal heat conduction on the effectiveness of the heat exchanger depends on the NTU-values, heat transfer coefficients and two dimensionless

groups $K_h = \frac{kS_h}{\dot{m}_h C_{p_h} L}$, $K_c = \frac{kS_c}{\dot{m}_c C_{p_c} L}$ (Eq (7.35) and Eq

(7.36)), as discussed in chapter 7.

From section 8.1 and the given mass flow rate range it is found that K_h and K_c will then be in the range from 0.10 to 1.70.

The calculation of the heat exchanger without longitudinal heat conduction is based on the NTU-concept of section 3.1, where the longitudinal heat conduction is neglected. For laminar flow, when heat is transferred from both sides of the channel, the Nusselt number has been calculated by means of the equation [23]

$$\left. \begin{aligned} Nu &= \left\{ Nu_{\infty}^3 + \left[f_g^3 + \sqrt{\frac{2 \cdot Re \cdot Pr \cdot \frac{d}{L}}{1 + 22 \cdot Pr}} \right] \cdot Re \cdot Pr \cdot \frac{d}{L} \right\}^{1/3} \\ Re &= \frac{\rho c d}{\mu} \end{aligned} \right\} \dots (8.2)$$

, where d is the hydraulic diameter $d = \frac{4S}{P}$ of a sub-channel.

$$Nu_{\infty} = 3.66 + \left[4 - \frac{0.102}{\left(\frac{d_i}{d_o}\right) + 0.02} \right] \cdot \left[\frac{d_i}{d_o} \right]^{0.04} \dots\dots\dots (8.3)$$

$$f_g = 1.615 \left[1 + 0.14 \left(\frac{d_i}{d_o}\right)^{0.1} \right] \dots\dots\dots (8.4)$$

d_i is the inner and d_o is the outer diameter of an annulus and the f_g factor is given by Stefan [22].

For turbulent flow the Nusselt number is given by [23]

$$Nu = \frac{\xi}{8} \cdot \frac{(Re - 1000)Pr}{1 + 12.7 \sqrt{\frac{\xi}{8}} (Pr^{2/3} - 1)} \left[1 + \left(\frac{d}{L}\right)^{2/3} \right] \dots\dots (8.5)$$

, where

$$\xi = \xi(Re) = 1.82 \cdot (10 \log Re - 1.64)^{-2} \dots\dots\dots (8.6)$$

for smooth tubes.

The Nusselt number for annular ducts is

$$\left. \begin{aligned} Nu_{\text{annulus}} &= Nu \cdot f\left(\frac{d_i}{d_o}\right) \\ \text{where} \\ f\left(\frac{d_i}{d_o}\right) &= 1 - 0.14 \left(\frac{d_i}{d_o}\right) \end{aligned} \right\} \dots\dots\dots (8.7)$$

for the case when heat is transferred from both sides of the channel.

The Nusselt number for turbulent flow is used if $Re > 2300$ and if the turbulent Nusselt number is higher than the laminar. Both Nusselt numbers are the average (not local) ones.

The effectiveness of the heat exchanger and the outlet temperatures have been calculated from the Eqs (3.1) through (3.7).

8.10 Effectiveness calculation applied to the tested heat exchanger

When evaluating the tests the data of section 8.1 are used, and also the fact that $\dot{m}_h \approx \dot{m}_c$, section 8.6.

Then from Eqs (7.35) and (7.36)

$$K_c = K_h = \frac{kS_h}{C_{p_h} \dot{m}_h L} \dots\dots\dots (8.8)$$

Since longitudinal heat conduction occurs both in the aluminum plates and in the brass rods

$$\begin{aligned} kS_h &= 225 \cdot 57 \cdot 0.2 \cdot 0.0003 + 100 \cdot 28 \cdot 5 \cdot 0.003 \cdot 0.003 = \\ &= 0.7695 + 0.1260 = 0.8955 \frac{\text{Wm}}{\text{K}} \dots\dots\dots (8.9) \end{aligned}$$

Thus

$$K_c = K_h = \frac{0.8955}{1005 \cdot 0.2 \cdot \dot{m}_h} = \frac{0.004455}{\dot{m}_h} \dots\dots\dots (8.10)$$

For each $\dot{m}_h$ it is thus possible to compute $K_c = K_h$ and $NTU = \frac{UA}{(\dot{m}C_p)_h}$ and also from chapter 7 the effectiveness without and with longitudinal conduction.

Assume that the following parameters are given.

Mass flow rate throughout heat exchanger

$$\dot{m} = \dot{m}_c = \dot{m}_h = 0.0160 \text{ kg/s}$$

$$T_{h_{in}} = 50^\circ\text{C}$$

$$T_{c_{in}} = 20^\circ\text{C}$$

Pressure assumed uniform: 1 bar

From data earlier given in section 8.1 on this particular tested heat exchanger the cross-sectional area for the fluid on each side is

$$S = 0.01554 \text{ m}^2$$

The mass velocity on each side will then be

$$\rho c = \frac{\dot{m}}{S} = \frac{0.0160}{0.01554} = 1.030 \frac{\text{kg}}{\text{sm}^2}$$

Heat transfer area (section 8.1) $A = 2.20 \text{ m}^2$

The effectiveness of the heat exchanger is assumed to be in the range of $\epsilon = \eta_h = \eta_c = 0.6$ (Eqs (3.1) through (3.7)). From this assumption the properties of the air are determined at the average temperature of each side.

Thus by means of Eqs (3.12) and (3.13)

$$\text{Hot side: } \frac{T_{h1} + T_{h2}}{2} = T_{h1} - \frac{\epsilon(T_{h1} - T_{c1})}{2} =$$

$$= 50 - \frac{0.6 \cdot 30}{2} = 41^\circ\text{C}$$

$$\text{Cold side: } \frac{T_{c1} + T_{c2}}{2} = T_{c1} + \frac{\epsilon(T_{h1} - T_{c1})}{2} =$$

$$= 20 + \frac{0.6 \cdot 30}{2} = 29^\circ\text{C}$$

The fluid properties will be determined at 40°C and 30°C because of the weak influence of the temperatures upon these properties.

From [27, chapter D6]

$$\mu_h = 18.81 \cdot 10^{-6} \frac{\text{kg}}{\text{ms}}$$

$$\mu_c = 18.40 \cdot 10^{-6} \frac{\text{kg}}{\text{ms}}$$

$$k_h = 27.49 \cdot 10^{-3} \frac{\text{W}}{\text{mK}}$$

$$k_c = 26.76 \cdot 10^{-3} \frac{\text{W}}{\text{mK}}$$

$$C_p = 1007 \frac{\text{J}}{\text{kgK}}$$

$$\text{Pr}_c \approx \text{Pr}_h \approx 0.70$$

The hydraulic diameter (section 8.1) is

$$d \approx 5.63 \text{ mm}$$

The Reynolds numbers are

$$\text{Re}_h = \left(\frac{\rho c d}{\mu} \right)_h = \frac{1.03 \cdot 5.63 \cdot 10^{-3}}{18.81 \cdot 10^{-6}} \approx 309$$

$$\text{Re}_c = \left(\frac{\rho c d}{\mu} \right)_c = \frac{1.03 \cdot 5.63 \cdot 10^{-3}}{18.40 \cdot 10^{-6}} \approx 316$$

From this information the Nusselt number can be calculated from Eqs (8.2) through (8.4)

$$\text{Nu} = \left\{ \text{Nu}_\infty^3 + \left[f_g^3 + \sqrt{\frac{2 \cdot \text{Re} \cdot \text{Pr} \cdot \frac{d}{L}}{1 + 22 \cdot \text{Pr}}} \right] \cdot \text{Re} \cdot \text{Pr} \cdot \frac{d}{L} \right\}^{1/3}$$

where

$$\text{Nu}_\infty = 3.66 + \left[4 - \frac{0.102}{\left(\frac{d_i}{d_o} \right) + 0.02} \right] \cdot \left[\frac{d_i}{d_o} \right]^{0.04}$$

$$f_g = 1.615 \left[1 + 0.14 \left(\frac{d_i}{d_o} \right)^{0.1} \right]$$

The value $\frac{d_i}{d_o} = 1$ for a plane channel.

$$\text{Nu}_\infty = 3.66 + \left[4 - \frac{0.102}{1 + 0.02} \right] = 7.56$$

$$f_g = 1.615(1 + 0.14) = 1.841$$

For the hot side the result is

$$Nu_h = \left(\frac{hd}{k}\right)_h = \left\{ 7.56^3 + \left[1.841^3 + \sqrt{\frac{2 \cdot 309 \cdot 0.70 \cdot \frac{5.63}{200}}{1 + 22 \cdot 0.70}} \cdot 309 \cdot 0.7 \cdot \frac{5.63}{200} \right] \right\}^{1/3}$$

$$Nu_h = 7.80$$

In the same way

$$Nu_c = 7.81$$

$$h_h = \frac{Nu_h \cdot k_h}{d} = \frac{7.80 \cdot 27.49 \cdot 10^{-3}}{5.63 \cdot 10^{-3}} = 38.09 \frac{W}{m^2 K}$$

$$h_c = \frac{Nu_c \cdot k_c}{d} = \frac{7.81 \cdot 26.76 \cdot 10^{-6}}{5.63 \cdot 10^{-3}} = 37.12 \frac{W}{m^2 K}$$

From Eq (1.2)

$$\frac{1}{U} = \frac{1}{h_c} + \frac{1}{h_h} = \frac{1}{37.12} + \frac{1}{38.09} = 0.0532$$

$$U = 18.80 \frac{W}{m^2 K}$$

Eq (3.7) gives

$$NTU = \frac{UA}{(\dot{m}C_p)_c} = \frac{UA}{(\dot{m}C_p)_h} = \frac{18.80 \cdot 2.20}{1.6 \cdot 10^{-2} \cdot 1007} = 2.57$$

$$\frac{C_c}{C_h} = 1$$

Eqs (3.1) through (3.7) or Fig 35 give

$$\epsilon_{\text{no conduction}} = \eta_c = \eta_h = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = 0.655$$

From Eq (8.10)

$$K_h = K_c = \frac{k \cdot S_h}{C_{p_h} \cdot \dot{m}_h \cdot L} = \frac{0.004455}{\dot{m}} = \frac{0.004455}{0.016} = 0.2784$$

From Eqs (7.55) and (7.56)

$$Y_h = Y_c = \frac{K_h \cdot C_h}{NTU \cdot C_{\min}} = \frac{0.2784}{2.57} = 0.1083$$

From Fig 36 $\Delta\epsilon_{rel} = 0.118$.

From Eq (7.58)

$$\epsilon_{with\ condensation} = \epsilon_{no\ condensation}(1 - \Delta\epsilon_{rel}) = 0.655(1 - 0.118)$$

$$\epsilon_{with\ condensation} \approx 0.578$$

The results of similar calculations are given in Table 8.10.1 and are also found in Fig 62a, b and c as one of the two curves.

Table 8.10.1 Calculated values for tested heat exchanger

$\dot{m}$ (kg/s)	$K_c = K_h$	$Y_c = Y_h$	NTU	$\epsilon_{no\ condensation}$	$\epsilon_{with\ condensation}$
0.0412	0.108	0.108	1	0.476	0.463
0.0196	0.227	0.114	2	0.614	0.563
0.0126	0.354	0.118	3	0.681	0.586
0.0094	0.464	0.116	4	0.722	0.589
0.0076	0.586	0.117	5	0.750	0.585
0.0064	0.696	0.116	6	0.772	0.579
0.0057	0.782	0.112	7	0.789	0.575
0.0050	0.891	0.111	8	0.802	0.570
0.0045	0.990	0.110	9	0.813	0.565
0.0040	1.110	0.111	10	0.823	0.560

8.11 Test evaluation procedure

When the pressure of the vapor in the air p_v is lower than the vapor saturation pressure p_g , no condensation will occur and the pressure p_v is constant throughout the test rig and heat exchanger. From the two relations

$$\left. \begin{aligned} p_v &= \phi \cdot p_g \\ p_g &= f(T) \end{aligned} \right\} \dots\dots\dots(8.11)$$

where ϕ is the relative humidity and from the temperature measurement where the humidity probe was located (Fig 30)

the vapour pressure was calculated by means of the function for p_g [21, p 32]. The following relations can then be established if the total pressure p_{tot} is assumed to be the barometric pressure.

The dry air pressure

$$p_a = p_{tot} - p_v \quad \dots\dots\dots(8.12)$$

$$\rho_v = \frac{p_v}{R_v \cdot T} \quad \dots\dots\dots(8.13)$$

$$\rho_a = \frac{p_a}{R_a \cdot T} \quad \dots\dots\dots(8.14)$$

$$\rho_{tot} = \rho_v + \rho_a \quad \dots\dots\dots(8.15)$$

The humidity ratio

$$\omega = \frac{\rho_v}{\rho_a} \quad \dots\dots\dots(8.16)$$

The maximum velocity in the measurement tubes is

$$c_{max} = \sqrt{\frac{2p_{dyn}}{\rho_{tot}}} \quad \dots\dots\dots(8.17)$$

as mentioned in section 8.3. The average velocity c (Fig 32b) can now be calculated by means of an iteration procedure which is based on the following formulas. Calculate the Reynolds number

$$Re = \frac{\rho_{tot} \cdot c \cdot D}{\mu} \quad \dots\dots\dots(8.18)$$

, where $D = 16.3$ mm is the inner diameter of the measurement tube and [25]

$$\mu = (18.18 + 0.0495(T - 293.15))10^{-6} \frac{kg}{sm} \quad \dots\dots\dots(8.19)$$

$$\left. \begin{aligned} \frac{c}{c_{max}} &= 0.1653 \ln (10 \log Re) + 0.5655 \\ 7000 < Re < 30\,000 \end{aligned} \right\} \dots\dots\dots(8.20a)$$

$$\left. \begin{aligned} \frac{c}{c_{\max}} &= 0.57 \ln (10^{\log Re}) + 6.36 \cdot 10^{-3} \\ 2500 &\leq Re \leq 7000 \end{aligned} \right\} \dots\dots\dots (8.20b)$$

$$\left. \begin{aligned} \frac{c}{c_{\max}} &= e^{2.8 \cdot 10^{\log Re} \cdot 4.6484 \cdot 10^{-5}} \\ 2050 &< Re < 2500 \end{aligned} \right\} \dots\dots\dots (8.20c)$$

and

$$\left. \begin{aligned} \frac{c}{c_{\max}} &= 0.5 \\ Re &< 2050 \end{aligned} \right\} \dots\dots\dots (8.20d)$$

Because of the behaviour of the curve of Fig 32b near the critical Reynolds number, the following iteration procedure is used.

The iteration procedure has the following steps.

1. A Reynolds number based on $c_{\max}$ is calculated.
2. $\frac{c}{c_{\max}}$ is calculated by means of Eq (8.20).
3. c is calculated.
4. A new Reynolds number based on c is calculated.
5. A new $\frac{c}{c_{\max}}$ is calculated.
6. This iteration ends when the difference in $\frac{c}{c_{\max}}$ is less than $5 \cdot 10^{-3}$.
7. The assumption is then made that $\frac{c}{c_{\max}} = 0.5$.
8. The steps 3 through 6 are performed once more.
9. c is taken as the average of the two velocities.

This is done for all 32 measurement tubes of Fig 30.

The mass flow rate can be calculated in each tube in the following way

$$\dot{V} = c \cdot S_{\text{tube}} \dots\dots\dots (8.21)$$

$$\dot{m}_a = \rho_a \cdot \dot{V} \quad \dots\dots\dots (8.22)$$

$$\dot{m}_v = \rho_v \cdot \dot{V} \quad \dots\dots\dots (8.23)$$

$$\dot{m} = \dot{m}_{tot} = \dot{m}_v + \dot{m}_a \quad \dots\dots\dots (8.24)$$

The specific enthalpy for air is

$$\left. \begin{aligned} i &= C_{p_a} \cdot T + \omega(r + C_{p_v} \cdot T) \frac{J}{kg \text{ dry air}} \\ , \text{ where from [26]} \\ C_{p_a} &= 1000 \frac{J}{kgK} \\ C_{p_v} &= 1860 \frac{J}{kgK} \\ r &= 2500 \cdot 10^3 \frac{J}{kg} \end{aligned} \right\} \dots\dots (8.25)$$

The heat flow rate from the hot to the cold side can then be calculated

$$q_h = \sum_1^{16} \Delta i_h \dot{m}_{a_h} \quad \dots\dots\dots (8.26)$$

or

$$q_c = \sum_{19}^{34} \Delta i_c \dot{m}_{a_c} \quad \dots\dots\dots (8.27)$$

The relative_error in the heat balance is defined as

$$\frac{\Delta q}{q} = \frac{q_h - q_c}{q_h} \quad \dots\dots\dots (8.28)$$

Eq (3.7) gives

$$\left. \begin{aligned} \epsilon &= \frac{q}{C_{min}(T_{h1} - T_{c1})} \\ q &= C_h(T_{h1} - T_{h2}) = C_c(T_{c2} - T_{c1}) \end{aligned} \right\} \dots\dots\dots (8.29)$$

For non-uniform inlet temperature and mass velocity profiles the following equations (Compare Eqs (3.27) through (3.31).) may be used for the case

$$C_{min} = C_c \text{ and } C_p = \text{constant}$$

$$\left. \begin{aligned}
 q &= \int_{\dot{m}_c} C_p (T_{c_2} - T_{c_1}) d\dot{m}_c \\
 C &= \int_{\dot{m}} C_p d\dot{m} \\
 \bar{T}_{h_1} &= \frac{1}{\dot{m}_h} \int T_{h_1} d\dot{m}_h \\
 \bar{T}_{c_1} &= \frac{1}{\dot{m}_c} \int T_{c_1} d\dot{m}_c
 \end{aligned} \right\} \dots\dots\dots (8.30)$$

Thus

$$\epsilon = \frac{\int_{\dot{m}_c} C_p (T_{c_2} - T_{c_1}) d\dot{m}_c}{\left[\frac{1}{\dot{m}_h} \int T_{h_1} d\dot{m}_h - \frac{1}{\dot{m}_c} \int T_{c_1} d\dot{m}_c \right] \int_{\dot{m}_c} C_p d\dot{m}_c} \dots\dots\dots (8.31)$$

If the average temperature

$$\bar{T} = \frac{1}{\dot{m}} \int T d\dot{m} \dots\dots\dots (8.32)$$

then

$$\epsilon = \frac{\bar{T}_{c_2} - \bar{T}_{c_1}}{\bar{T}_{h_1} - \bar{T}_{c_1}} \dots\dots\dots (8.33)$$

The average inlet and outlet temperatures on the hot and cold sides were calculated in the following way (Eq (8.32))

$$\bar{T} = \frac{\Sigma T \cdot \dot{m}}{\Sigma \dot{m}} \dots\dots\dots (8.34)$$

and from this value of the temperatures, $\epsilon = \eta_c = \eta_h$ (Eqs (3.5) through (3.6) and (8.33)) were calculated for the case $\dot{m}_h = \dot{m}_c$. The test results were calculated (on line) during the measurements by the computer. An example is given in Tables 8.11.1 and 8.11.2.

Table 8.11.1 Primary results from test No. 36 [29]

Isothermal box: 20.75°C

Barometric pressures: 756 mm Hg

Temperatures

Hot inlet air (1-16) (°C)

51.51	50.93	51.12	50.54
52.54	52.29	51.61	50.93
52.49	51.95	51.27	50.98
52.39	51.78	51.42	50.98

Hot outlet air (37-52) (°C)

35.21	35.60	35.50	34.81
37.79	37.30	37.06	36.96
39.36	39.11	38.57	37.45
40.19	39.94	39.48	38.72

Cold inlet air (19-34) (°C)

23.49	22.56	24.22	24.07
22.27	22.22	22.46	22.46
22.17	22.22	22.17	22.17
22.22	22.17	22.22	22.12

Cold outlet air (55-70) (°C)

38.43	39.60	39.50	39.75
36.43	37.26	37.79	38.13
34.57	35.60	35.84	36.13
33.25	34.08	34.47	34.81

Dynamic pressures

Hot side (1-16) (mm Wg)

7.73	7.71	7.75	7.75
7.78	7.76	7.80	7.81
7.71	7.81	7.85	7.80
7.78	7.79	7.76	7.90

Cold side (19-34) (mm Wg)

7.72	7.73	7.78	7.84
7.71	7.73	7.76	7.73
7.70	7.72	7.76	7.60
7.58	7.62	7.58	7.58

Each tube was checked during the test. Two examples are given below.

Tube_1_(hot)

$$p_{g_1} = 13289 \text{ Pa}$$

$$p_{g_2} = 5684 \text{ Pa}$$

$$\frac{c}{c_{\max}} = 0.7926$$

$$c = 9.16 \text{ m/s}$$

$$c_{\max} = 11.56 \text{ m/s}$$

$$Re = 8958$$

$$\rho_a = 1.126 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_v = 0.0094 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_{\text{tot}} = 1.136 \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 18.93 \cdot 10^{-6} \frac{\text{kg}}{\text{sm}}$$

$$\dot{V} = 0.001915 \text{ m}^3/\text{s}$$

$$i_{h_1} = 73208 \frac{\text{J}}{\text{kg}}$$

$$i_{h_2} = 56646 \frac{\text{J}}{\text{kg}}$$

$$\phi_{h_1} = 10.09 \%$$

$$\phi_{h_2} = 23.09 \%$$

Tube_19_(cold)

$$p_{g_1} = 2890 \text{ Pa}$$

$$p_{g_2} = 6777 \text{ Pa}$$

$$\frac{c}{c_{\max}} = 0.7923$$

$$c = 9.20 \text{ m/s}$$

$$c_{\max} = 11.61 \text{ m/s}$$

$$Re = 8821$$

$$\rho_a = 1.113 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_v = 0.0104 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_{\text{tot}} = 1.123 \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 19.09 \cdot 10^{-6} \frac{\text{kg}}{\text{sm}}$$

$$\dot{V} = 0.001922 \text{ m}^3/\text{s}$$

$$i_{c_1} = 47209 \frac{\text{J}}{\text{kg}}$$

$$i_{c_2} = 62410 \frac{\text{J}}{\text{kg}}$$

$$\phi_{c_1} = 51.69 \%$$

$$\phi_{c_2} = 22.04 \%$$

From the calculations performed on each tube the final calculations were performed, Table 8.11.2.

Table 8.11.2 Heat exchanger test No. 36 [29]

Test			
Hot inlet air (1-16) ($^{\circ}\text{C}$)			
51.51	50.93	51.12	50.54
52.54	52.29	51.61	50.93
52.49	51.95	51.27	50.98
52.39	51.78	51.42	50.98
Hot outlet air (37-52) ($^{\circ}\text{C}$)			
35.21	35.60	35.50	34.81
37.79	37.30	37.06	36.96
39.36	39.11	38.57	37.45
40.19	39.94	39.48	38.72
Cold inlet air (19-34) ($^{\circ}\text{C}$)			
23.49	22.56	24.22	24.07
22.27	22.22	22.46	22.46
22.17	22.22	22.17	22.17
22.22	22.17	22.22	22.12
Cold outlet air (55-70) ($^{\circ}\text{C}$)			
38.43	39.60	39.50	39.75
36.43	37.26	37.79	38.13
34.57	35.60	35.84	36.13
33.25	34.08	34.47	34.81

$$\bar{T}_{h1} = 51.54^{\circ}\text{C}$$

$$\bar{T}_{h2} = 37.69^{\circ}\text{C}$$

$$\bar{T}_{c1} = 22.57^{\circ}\text{C}$$

$$\bar{T}_{c2} = 36.60^{\circ}\text{C}$$

$$\text{Hot mass flow rate} = 0.0347 \text{ kg/s}$$

$$\text{Cold mass flow rate} = 0.0346 \text{ kg/s}$$

$$\text{Capacity ratio} = 0.996$$

$$\eta_h = 0.478$$

$$\epsilon = \eta_c = 0.484$$

$$\frac{\Delta q}{q_h} = -0.915 \%$$

In Fig 62a, b, c [29] and in Table 8.12.1 the results from the calculations with and without longitudinal conduction (Table 8.10.1) are compared with measured values for the tested heat exchanger (section 8.1), and with air inlet temperatures of approximately 50°C and 20°C.

Fig 62a, b, c show very good agreement between calculated values with longitudinal conduction and measurements. The values differ at no point (except case c) more than about 2 % in ϵ . The measured values were selected under the following conditions:

- a. The difference between hot and cold mass flow rates is not more than 2 %.
- b. The error in heat balance (Eq (8.30)) is not more than 3 % (except in case c).
- c. A special case are the two lowest mass flow rates of about 0.0056 kg/s and 0.0030 kg/s. In these cases the heat balance differed between 6 % and 13 %, probably due to increased importance of heat losses (including conduction in the 32 inlet and outlet ducts of the heat exchanger) to the surroundings. It was found that η_h formed one group and η_c another group. The two points (circle with dot inside) for $\dot{m} = 0.0056$ kg/s and 0.0030 kg/s in Fig 62a, b, c were obtained as an average of all η_h and η_c .

Table 8.12.1 Test results on crossflow heat exchanger.

No.	$\dot{m}_h$ (kg/s)	$\dot{m}_c$ (kg/s)	NTU	$\frac{1}{NTU}$	η_h	η_c	ϵ	$\frac{\Delta q}{q}$ %
1	0.0440	0.0446	0.84	1.19	0.438	0.437	0.438	-0.979
2	0.0442	0.0446	0.83	1.20	0.438	0.439	0.439	-1.384
3	0.0442	0.0446	0.83	1.20	0.438	0.439	0.439	-1.315
4	0.0441	0.0446	0.83	1.20	0.438	0.440	0.440	-1.415
5	0.0441	0.0446	0.83	1.20	0.440	0.439	0.440	-1.068
6	0.0441	0.0445	0.84	1.19	0.439	0.440	0.440	-1.037
7	0.0441	0.0445	0.84	1.19	0.440	0.440	0.440	-1.220
8	0.0441	0.0446	0.84	1.19	0.440	0.440	0.440	-1.110
9	0.0441	0.0445	0.84	1.19	0.439	0.440	0.440	-1.199
10	0.0440	0.0445	0.85	1.18	0.439	0.441	0.441	-1.621
11	0.0440	0.0445	0.85	1.18	0.440	0.440	0.440	-1.088
12	0.0441	0.0444	0.85	1.18	0.440	0.441	0.441	-0.938
13	0.0440	0.0444	0.85	1.18	0.441	0.441	0.441	-0.869
14	0.0440	0.0443	0.86	1.16	0.441	0.441	0.441	-0.815
15	0.0440	0.0444	0.86	1.16	0.440	0.442	0.442	-1.267
16	0.0441	0.0444	0.85	1.18	0.441	0.440	0.441	-0.728
17	0.0392	0.0396	1.07	0.93	0.459	0.460	0.460	-1.021
18	0.0393	0.0396	1.07	0.93	0.459	0.460	0.460	-1.053
19	0.0394	0.0398	1.05	0.95	0.459	0.460	0.460	-1.246
20	0.0395	0.0399	1.05	0.95	0.460	0.458	0.460	-0.550
21	0.0394	0.0397	1.06	0.94	0.461	0.459	0.461	-0.381
22	0.0395	0.0399	1.05	0.95	0.459	0.459	0.459	-1.008
23	0.0392	0.0395	1.07	0.93	0.459	0.461	0.461	-1.277
24	0.0393	0.0394	1.07	0.93	0.460	0.460	0.460	-0.326
25	0.0391	0.0394	1.08	0.92	0.461	0.460	0.461	-0.541
26	0.0392	0.0395	1.07	0.93	0.460	0.460	0.460	0.728
27	0.0393	0.0394	1.07	0.93	0.461	0.459	0.461	-0.278
28	0.0392	0.0395	1.07	0.93	0.460	0.460	0.460	-0.782
29	0.0391	0.0393	1.08	0.92	0.460	0.461	0.461	-0.722
30	0.0392	0.0394	1.07	0.93	0.461	0.460	0.461	-0.265
31	0.0393	0.0394	1.07	0.93	0.458	0.459	0.459	-0.603
32	0.0393	0.0395	1.07	0.93	0.459	0.458	0.459	-0.300

Table 8.12.1 Test results on crossflow heat exchanger (cont.)

No.	$\dot{m}_h$ (kg/s)	$\dot{m}_c$ (kg/s)	NTU	$\frac{1}{NTU}$	η_h	η_c	ϵ	$\frac{\Delta q}{q}$ %
33	0.0343	0.0341	1.24	0.80	0.478	0.487	0.487	-1.562
34	0.0347	0.0346	1.22	0.82	0.478	0.484	0.484	-0.915
35	0.0346	0.0345	1.22	0.82	0.477	0.485	0.485	-1.583
36	0.0347	0.0344	1.23	0.81	0.477	0.484	0.484	-0.624
37	0.0347	0.0346	1.22	0.82	0.477	0.484	0.484	-1.231
38	0.0341	0.0337	1.26	0.79	0.476	0.486	0.486	-1.026
39	0.0338	0.0335	1.28	0.78	0.478	0.491	0.491	-1.799
40	0.0338	0.0336	1.28	0.78	0.481	0.490	0.490	-1.464
41	0.0339	0.0337	1.27	0.79	0.480	0.490	0.490	-1.818
42	0.0339	0.0336	1.27	0.79	0.480	0.490	0.490	-1.337
43	0.0338	0.0335	1.28	0.78	0.480	0.489	0.489	-1.090
44	0.0336	0.0337	1.28	0.78	0.481	0.491	0.491	-2.379
45	0.0296	0.0300	1.44	0.69	0.503	0.504	0.504	-1.493
46	0.0296	0.0297	1.45	0.69	0.504	0.502	0.504	-0.269
47	0.0297	0.0300	1.44	0.69	0.503	0.502	0.503	-0.761
48	0.0301	0.0303	1.42	0.70	0.502	0.502	0.502	-0.731
49	0.0302	0.0302	1.41	0.71	0.501	0.501	0.501	-0.109
50	0.0303	0.0304	1.40	0.71	0.501	0.501	0.501	-0.772
51	0.0297	0.0293	1.46	0.68	0.503	0.501	0.503	1.630
52	0.0294	0.0291	1.48	0.67	0.505	0.506	0.506	0.889
53	0.0290	0.0291	1.49	0.67	0.508	0.507	0.508	-0.255
54	0.0291	0.0291	1.49	0.67	0.507	0.506	0.507	-0.101
55	0.0291	0.0290	1.49	0.67	0.507	0.505	0.507	0.401
56	0.0291	0.0290	1.49	0.67	0.507	0.506	0.507	0.323
57	0.0289	0.0293	1.49	0.67	0.507	0.507	0.507	-1.257

Table 8.12.1 Test results on crossflow heat exchanger (cont.)

No.	$\dot{m}_h$ (kg/s)	$\dot{m}_c$ (kg/s)	NTU	$\frac{1}{NTU}$	η_h	η_c	ϵ	$\frac{\Delta q}{q}$ %
58	0.0250	0.0249	1.70	0.59	0.524	0.523	0.524	1.184
59	0.0256	0.0254	1.68	0.60	0.523	0.520	0.523	1.230
60	0.0259	0.0257	1.66	0.60	0.521	0.520	0.521	0.948
61	0.0256	0.0254	1.68	0.60	0.521	0.519	0.521	1.132
62	0.0247	0.0244	1.72	0.58	0.523	0.523	0.523	1.152
63	0.0245	0.0241	1.74	0.57	0.526	0.529	0.529	0.987
64	0.0246	0.0242	1.73	0.58	0.527	0.527	0.527	1.498
65	0.0246	0.0243	1.73	0.58	0.526	0.527	0.527	0.996
66	0.0246	0.0242	1.73	0.58	0.527	0.526	0.527	1.848
67	0.0246	0.0241	1.74	0.57	0.527	0.527	0.527	1.794
68	0.0247	0.0244	1.72	0.58	0.527	0.527	0.527	1.127
69	0.0208	0.0208	1.91	0.52	0.546	0.540	0.546	1.057
70	0.0211	0.0211	1.89	0.53	0.543	0.538	0.543	1.062
71	0.0210	0.0208	1.90	0.53	0.542	0.540	0.542	1.276
72	0.0212	0.0208	1.90	0.53	0.542	0.539	0.542	2.387
73	0.0206	0.0203	1.94	0.52	0.546	0.543	0.546	1.537
74	0.0204	0.0201	1.96	0.51	0.545	0.543	0.545	1.732
75	0.0204	0.0204	1.94	0.52	0.547	0.541	0.547	1.127
76	0.0206	0.0204	1.93	0.51	0.547	0.539	0.547	2.118
77	0.0204	0.0204	1.94	0.52	0.549	0.538	0.549	2.181
78	0.0204	0.0203	1.95	0.51	0.548	0.539	0.548	2.064
79	0.0204	0.0203	1.95	0.51	0.548	0.539	0.548	1.789
80	0.0207	0.0211	1.90	0.53	0.548	0.538	0.548	-0.007

Table 8.12.1 Test results on crossflow heat exchanger (cont.)

No.	$\dot{m}_h$ (kg/s)	$\dot{m}_c$ (kg/s)	NTU	$\frac{1}{NTU}$	η_h	η_c	ϵ	$\frac{\Delta q}{q}$ %
81	0.0154	0.0154	2.48	0.40	0.574	0.559	0.574	2.712
82	0.0155	0.0154	2.47	0.40	0.568	0.567	0.562	0.774
83	0.0156	0.0155	2.45	0.41	0.568	0.567	0.568	0.546
84	0.0154	0.0154	2.48	0.40	0.568	0.569	0.569	-0.242
85	0.0155	0.0154	2.47	0.40	0.567	0.570	0.570	0.208
86	0.0158	0.0156	2.42	0.41	0.565	0.569	0.569	0.164
87	0.0159	0.0156	2.42	0.41	0.561	0.574	0.574	-0.101
88	0.0158	0.0154	2.45	0.41	0.563	0.573	0.573	0.87
89	0.0153	0.0151	2.50	0.40	0.565	0.574	0.574	-0.156
90	0.0154	0.0151	2.50	0.40	0.566	0.572	0.572	0.609
91	0.0155	0.0152	2.48	0.40	0.566	0.573	0.573	0.349
92	0.0152	0.0149	2.52	0.39	0.566	0.575	0.575	0.368
93	0.0152	0.0149	2.52	0.39	0.566	0.574	0.574	0.099
94	0.0152	0.0150	2.52	0.39	0.566	0.574	0.574	-0.364
95	0.0153	0.0153	2.49	0.40	0.566	0.574	0.574	-1.499
96	0.0096	0.0095	3.97	0.25	0.586	0.583	0.586	1.334
97	0.0094	0.0093	4.03	0.25	0.583	0.584	0.584	1.031
98	0.0095	0.0095	3.97	0.25	0.583	0.585	0.585	-0.003
99	0.0098	0.0096	3.87	0.26	0.583	0.586	0.586	1.210
100	0.0095	0.0094	4.00	0.25	0.579	0.591	0.591	-0.940
101	0.0093	0.0092	4.07	0.25	0.582	0.586	0.586	0.697
102	0.0091	0.0091	4.15	0.24	0.583	0.585	0.585	1.669
103	0.0093	0.0092	4.07	0.25	0.583	0.585	0.585	1.121
104	0.0093	0.0092	4.07	0.25	0.584	0.584	0.584	1.045

Table 8.12.1 Test results on crossflow heat exchanger (cont.)

No.	$\dot{m}_h$ (kg/s)	$\dot{m}_c$ (kg/s)	NTU	$\frac{1}{NTU}$	η_h	η_c	ε	$\frac{\Delta q}{q}$ %
105	0.0056	0.0055	7.15	0.140	0.595	0.569	0.595	0.274
106	0.0057	0.0055	7.05	0.142	0.597	0.567	0.597	7.836
107	0.0056	0.0054	7.25	0.138	0.596	0.570	0.596	8.372
108	0.0057	0.0055	7.05	0.142	0.591	0.574	0.591	5.529
109	0.0057	0.0055	7.05	0.142	0.591	0.574	0.591	5.316
110	0.0057	0.0055	7.05	0.142	0.593	0.572	0.593	6.758
111	0.0055	0.0054	7.27	0.138	0.589	0.574	0.589	5.185
112	0.0054	0.0053	7.52	0.133	0.592	0.571	0.592	6.343
113	0.0055	0.0053	7.35	0.136	0.592	0.570	0.592	6.544
114	0.0055	0.0053	7.35	0.136	0.593	0.569	0.593	6.957
115	0.0055	0.0053	7.35	0.136	0.592	0.570	0.592	6.733
116	0.0056	0.0054	7.25	0.138	0.592	0.571	0.592	6.604
117	0.0056	0.0054	7.25	0.138	0.595	0.568	0.595	8.365
118	0.0056	0.0055	7.23	0.138	0.594	0.569	0.594	7.122
119	0.0056	0.0055	7.23	0.138	0.594	0.570	0.594	7.139
120	0.0057	0.0055	7.05	0.142	0.593	0.571	0.593	6.974
121	0.0029	0.0029	12.8	0.0781	0.580	0.529	0.580	9.839
122	0.0029	0.0029	12.8	0.0781	0.580	0.528	0.580	9.828
123	0.0028	0.0028	13.0	0.0769	0.569	0.539	0.569	6.828
124	0.0029	0.0028	12.9	0.0775	0.574	0.532	0.574	8.510
125	0.0029	0.0029	12.8	0.0781	0.584	0.524	0.584	11.816
126	0.0030	0.0029	12.75	0.0784	0.582	0.526	0.582	11.701
127	0.0029	0.0028	12.9	0.0775	0.578	0.527	0.578	10.132
128	0.0029	0.0029	12.8	0.0781	0.584	0.523	0.584	12.17
129	0.0029	0.0029	12.8	0.0781	0.581	0.526	0.581	10.863

8.13 Difference between results of calculations and measurements

The difference in Fig 62 between calculated and measured values is small. The difference may be due to

- a. The actual dimensions of the brass rods and the aluminium plates (including channel heights) may differ slightly from the nominal ones.
- b. The conductivity of the metals used may not be quite correct.
- c. The value of the Nusselt number used may differ slightly from the actual one.
- d. Small variations of the mass flow rate between outlet ducts and also between the seven subchannels of each outlet duct.
- e. Systematic errors in the measurements.

Among these five points a, b, c are considered most likely to have given the small difference between calculated and measured values. The effect on effectiveness due to variation in height is treated in [24].

8.14 Error analysis

Kline and McClintock [28] have shown that the way to combine the individual effect, which most nearly preserves the true statistical probability in an error analysis is to use a Root-Sum-Square addition

$$\delta R = \left[\left(\frac{\partial R}{\partial x_1} \delta x_1 \right)^2 + \left(\frac{\partial R}{\partial x_2} \delta x_2 \right)^2 + \dots + \left(\frac{\partial R}{\partial x_n} \delta x_n \right)^2 \right]^{1/2} \quad (8.35)$$

The error analysis can be divided into two parts, the temperature measurements and the mass flow rate measurements.

$$\dot{m} = \sum_{i=1}^{16} \dot{m}_i \quad \dots \dots \dots (8.36)$$

$$\dot{m}_i = (\rho_{\text{tot}} \cdot c \cdot S)_i \quad \dots \dots \dots (8.37)$$

$$\rho_{\text{tot}} = \rho_a + \rho_v \dots\dots\dots (8.38)$$

$$\rho_a \gg \rho_v$$

$$\rho_a = \frac{p_a}{R_a T} \dots\dots\dots (8.39)$$

$$p_a \approx p_{\text{tot}} \dots\dots\dots (8.40)$$

$$c = \sqrt{\frac{p_{\text{dyn}} \cdot 2}{\rho_{\text{tot}}}} \approx \sqrt{\frac{p_{\text{dyn}} \cdot 2}{\rho_a}} \dots\dots\dots (8.41)$$

$$c \cdot \rho_a \approx \rho_a \sqrt{\frac{p_{\text{dyn}} \cdot 2}{\rho_a}} \dots\dots\dots (8.42)$$

For one outlet tube

$$\dot{m}_i = (c \cdot \rho \cdot S)_i = \left[\sqrt{\frac{p_{\text{dyn}} \cdot 2 \cdot p_a}{R_a \cdot T}} \cdot S \right]_i \dots\dots\dots (8.43)$$

Eqs (8.35) and (8.43) give

$$\frac{\dot{m}_i}{\dot{m}_1} = \left[\left(\frac{\delta p_{\text{dyn}}}{2 p_{\text{dyn}}} \right)^2 + \left(\frac{\delta p_a}{2 p_a} \right)^2 + \left(\frac{\delta T}{2 T} \right)^2 + \left(\frac{\delta S}{S} \right)^2 \right]_i^{1/2} \dots\dots (8.44)$$

Assume the following relative errors for each i

$$\left. \begin{aligned} \frac{\delta p_{\text{dyn}}}{p_{\text{dyn}}} &\approx 0.02 \\ \frac{\delta p_a}{p_a} &\approx \frac{5}{760} = 0.0066 \\ \frac{\delta T}{T} &= \frac{1}{300} = 0.0033 \\ \frac{\delta S}{S} &= 0.01 \end{aligned} \right\} \dots\dots\dots (8.45)$$

Eqs (8.44) and (8.45) give

$$\frac{\delta \dot{m}_i}{\dot{m}_i} \approx 0.014 \quad \dots\dots\dots (8.46)$$

The result, Eq (8.46), shows that the error in one tube is 0.014, but in the experiment 16 tubes are used with the same error in each tube. Thus the error for the whole mass flow rate on one side is from Eqs (8.35) and (8.46)

$$\frac{\delta \dot{m}}{\dot{m}} = \sqrt{16} \cdot \frac{\delta \dot{m}_i}{\dot{m}_i} = 0.0573 \quad \dots\dots\dots (8.47)$$

About ten measurements have been performed on each mass flow rate $\dot{m}_i$ and the average sum of this value is used. Thus

$$\frac{\delta \dot{m}}{\dot{m}} = \frac{1}{\sqrt{10}} \cdot 0.0573 = 0.018 \quad \dots\dots\dots (8.48)$$

The temperature efficiency η_h on the hot side and η_c on the cold side is given by Eqs (3.5) and (3.6)

$$\eta_h = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}} \quad \dots\dots\dots (8.49)$$

$$\eta_c = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \quad \dots\dots\dots (8.50)$$

In all the measurements $\dot{m}_c \approx \dot{m}_h$. Thus

$$\left. \begin{aligned} \eta = \eta_h = \eta_c &= \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \\ 1 - \eta &= \frac{T_{h1} - T_{c2}}{T_{h1} - T_{c1}} \end{aligned} \right\} \dots\dots\dots (8.51)$$

Eqs (8.35) and (8.51) give

$$\delta\eta = \left[\left(\frac{\partial\eta}{\partial T_{c_1}} \delta T_{c_1} \right)^2 + \left(\frac{\partial\eta}{\partial T_{c_2}} \delta T_{c_2} \right)^2 + \left(\frac{\partial\eta}{\partial T_{h_1}} \delta T_{h_1} \right)^2 \right]^{1/2} \quad \dots (8.52)$$

$$\delta\eta = \frac{1}{T_{h_1} - T_{c_1}} \left[((1 - \eta) \delta T_{c_1})^2 + \delta T_{c_2}^2 + (\eta \delta T_{h_1})^2 \right]^{1/2} \quad \dots (8.53)$$

$$\frac{\delta\eta}{\eta} = \frac{1}{T_{h_1} - T_{c_1}} \left[\left(\frac{1 - \eta}{\eta} \delta T_{c_1} \right)^2 + \left(\frac{\delta T_{c_2}}{\eta} \right)^2 + (\delta T_{h_1})^2 \right]^{1/2} \quad \dots (8.54)$$

The following assumptions are made

$$\left. \begin{array}{l} \delta T = 0.4 \text{ K} \\ T_{h_1} - T_{c_1} = 30 \text{ K} \\ \eta \approx 0.50 \end{array} \right\} \dots \dots \dots (8.55)$$

In the case of temperature (and efficiency) measurements the error has not been assumed to be smaller because ten measurements are performed.

Eqs (8.54) and (8.55) give

$$\frac{\delta\eta}{\eta} = \frac{0.4}{30} \left[\left(\frac{0.5}{0.5} \right)^2 + \left(\frac{1}{0.5} \right)^2 + 1^2 \right]^{1/2} \quad \dots \dots \dots (8.56)$$

$$\frac{\delta\eta}{\eta} \approx 0.033 \quad \dots \dots \dots (8.57)$$

From these calculations it can be seen that the error in $\frac{\delta\eta}{\eta}$ is in the same range as $\frac{\delta\dot{m}}{\dot{m}} = 0.018$ (Eq (8.48)).

In Table 8.14.1 results for different mass flow rate are given. δp_{dyn} depends on the chosen range of the micro-manometer.

Table 8.14.1 Errors in mass flow rates

$\dot{m}$	$c_{\max}$	p_{dyn}	δp_{dyn}	$\frac{\delta \dot{m}_i}{\dot{m}_i}$	$\frac{\delta \dot{m}}{\dot{m}}$	
(kg/s)	($\frac{\text{m}}{\text{s}}$)	(Pa)	(Pa)		one	ten
					measurement	(s)
0.0446	14.0	117.6	3	0.0166	0.0664	0.0210
0.0396	12.6	95.26	1	0.0119	0.0476	0.0151
0.0350	11.2	75.26	1	0.0126	0.0504	0.0159
0.0300	9.8	57.62	1	0.0137	0.0548	0.0173
0.0250	8.4	42.34	1	0.0159	0.0636	0.0201
0.0200	7.0	29.40	0.3	0.0118	0.0472	0.0149
0.0152	5.6	18.82	0.3	0.0133	0.0532	0.0168
0.0095	4.2	10.58	0.3	0.0177	0.0708	0.0224
0.0055	2.8	4.70	0.5	0.0542	0.2168	0.0686
0.0028	1.4	1.18	0.5	0.2121	0.8484	0.2683

The errors in temperature efficiency have also been calculated by means of Eqs (8.55) and (8.57). The result is

$$\delta \eta = 0.0163 \text{ to } 0.0164 \quad \dots\dots\dots (8.58)$$

The errors in $\epsilon = \eta$ are shown in Fig 62a.

The error in heat flow rate (no condensation) between the hot and cold fluids can now be calculated.

For hot side

$$q_h = C_p \cdot \dot{m}_h (T_{h1} - T_{h2}) = C_p \cdot \dot{m}_h \cdot \Delta T_h \quad \dots\dots\dots (8.59)$$

For cold side

$$q_c = C_p \cdot \dot{m}_c (T_{c2} - T_{c1}) \quad \dots\dots\dots (8.60)$$

From Eqs (8.59) and (8.60)

$$\Delta q = q_h - q_c = C_p \cdot \dot{m}_h (T_{h1} - T_{h2}) - C_p \cdot \dot{m}_c (T_{c2} - T_{c1}) \quad (8.61)$$

Eqs (8.35) and (8.61) give

$$\frac{\delta q_h}{q_h} = \left\{ \left[\frac{\delta \dot{m}_h}{\dot{m}_h} \right]^2 + \left[\frac{\delta T_{h1}}{T_{h1} - T_{h2}} \right]^2 + \left[- \frac{\delta T_{h2}}{T_{h1} - T_{h2}} \right]^2 \right\}^{1/2} =$$

$$= \left\{ \left[\frac{\delta \dot{m}_h}{\dot{m}_h} \right]^2 + 2 \left[\frac{\delta T_h}{T_{h1} - T_{h2}} \right]^2 \right\}^{1/2} \dots\dots\dots (8.62)$$

and similarly for $\frac{\delta q_c}{q_c}$. Thus Eq (8.35), (8.61) and (8.62) give

$$\frac{\delta q}{q} = \sqrt{2} \cdot \frac{\delta q_h}{q_h} = \sqrt{2} \cdot \left\{ \left[\frac{\delta \dot{m}_h}{\dot{m}_h} \right]^2 + 2 \left[\frac{\delta T_h}{T_{h1} - T_{h2}} \right]^2 \right\}^{1/2} \dots (8.63)$$

The calculations have been performed for the same ten cases as in Table 8.14.1 under the assumption that

$$T_{h1} - T_{h2} = 15 \text{ K}$$

The thermocouples differed not more than 0.15 K from the standard polynomial (section 8.2) before the heat exchanger test began, but this error may become larger during these tests. Therefore four values of δT have been assumed

$$\delta T = 0.2, 0.25, 0.3 \text{ and } 0.4 \text{ K}$$

when the errors in the heat flow rates have been calculated in Table 8.14.2 (compare Table 8.14.1).

Table 8.14.2 Error in heat balance

			$\frac{\delta q}{q} = \sqrt{2} \cdot \frac{\delta q_h}{q_h}$			
$\dot{m}$ (kg/s)	$c_{\max}$ (m/s)	$\frac{\delta \dot{m}}{\dot{m}}$ (kg/s)	$\delta T = 0.2$ (K)	$\delta T = 0.25$ (K)	$\delta T = 0.3$ (K)	$\delta T = 0.4$ (K)
0.0446	14.0	0.0210	0.0399	0.0446	0.0498	0.0610
0.0396	12.6	0.0151	0.0342	0.0396	0.0453	0.0575
0.0350	11.2	0.0159	0.0349	0.0402	0.0459	0.0579
0.0300	9.8	0.0173	0.0362	0.0413	0.0469	0.0587
0.0250	8.4	0.0201	0.0399	0.0438	0.0491	0.0604
0.0200	7.0	0.0149	0.0340	0.0394	0.0452	0.0573
0.0152	5.6	0.0168	0.0357	0.0409	0.0465	0.0584
0.0095	4.2	0.0224	0.0414	0.0460	0.0510	0.0620
0.0055	2.8	0.0686	0.1006	0.1026	0.1049	0.1107
0.0028	1.4	0.2683	0.3804	0.3809	0.3815	0.3832

If $\frac{\delta q}{q}$ from Table 8.14.2 is compared with the relative heat balance error of Table 8.12.1 it is seen that

$$\frac{\Delta q}{q} < \frac{\delta q}{q} \dots\dots\dots (8.64)$$

8.15

Conclusions

The results of both tests and calculations show that the longitudinal heat conduction has to be taken into consideration when calculations are performed on a gas to gas crossflow heat exchanger. The lower mass flow rates used in the present test are of no practical interest but they show the great influence of conduction.

The question of how to avoid the disadvantage of longitudinal conduction will now be discussed. From the dimensionless groups ($\ell = L$) $K = \frac{k\delta}{C}$ (Eqs (7.35) and (7.36)) and $Y = \frac{k\delta}{UL^2}$ (Eqs (7.55) and (7.56)) the following conclusions can be drawn.

- a. Try to have as thin plates as possible, but then the pressure difference between the two air streams must be kept lower. Thin plates may also fail sooner due to corrosion.
- b. Keep the dimensions (high L) of the heat exchanger plates as large as possible.
- c. Choose a material with low conductivity (but compare Eqs (1.1) and (1.2)).
- d. If the mass flow rates are low, use fewer channels. Sometimes effectiveness increases (Fig 62) when $\dot{m}$ per channel increases although NTU decreases.

It is found that the relative error in the heat balance is less than what the corresponding error analysis predicts.

The first part of this work contains different mathematical models for calculating the effectiveness of thin-walled plate crossflow heat exchangers operating with gases on both sides, either in single or in multipass arrangements. In all cases the calculations were performed on the assumption that no mixing takes place within each pass. Between the different passes calculations were performed both with and without mixing. Calculations were performed for up to four passes. In the first part of the work the primary interest was to find as convenient calculation models as possible and to compare them with other published models. One important assumption was that no longitudinal heat conduction occurred in the heat exchanger wall. Some calculations (section 3.5) have been performed to show the influence of non-uniform inlet temperature and mass velocity profiles.

Six different methods were discussed concerning the calculation of a single crossflow heat exchanger. Method 3 (section 4.4.3) was considered to be the best one to use if both the temperature profiles and the effectiveness are of interest. Method 3 can also be used for non-uniform inlet temperature and mass velocity profiles. Method 3 was also used for countercurrent arrangements up to four passes (section 6.4) and the results showed very good agreement with the results of Sahlberg [8].

In the second part of this work the same calculations were performed for the case when longitudinal heat conduction in the heat exchanger wall was included. The calculations showed that the effectiveness of a gas-to-gas crossflow heat exchanger can be strongly dependant upon the longitudinal heat conduction in the wall. The results also showed that a multipass countercurrent arrangement reduces the influence of longitudinal heat conduction (sections

7.11.1 and 7.11.2) when compared with a single pass heat exchanger of the same no conduction effectiveness. .
 A remarkable result is that when NTU goes to infinity and K_h and K_c (Eqs (7.35) and (7.36)) also go to infinity (Eq (7.63))

$$\eta_c = \frac{C_h}{C_c + C_h}$$

if $T_{in} = 1$ and $t_{in} = 0$.

The dimensionless group Y (Eqs (7.55) and (7.56)), which is a constant in the laminar regime, is an important parameter. It is possible to calculate the effectiveness of a heat exchanger more accurately if Y together with the NTU-value and capacity ratio are known.

In the third part of this work results from the calculations without and with longitudinal heat conduction in the wall were compared with test results from a small crossflow heat exchanger. The test results showed good agreement with the results from the calculations with longitudinal heat conduction. The error analysis regarding heat balance also showed good agreement with the test results as soon as steady state was obtained.

The following conclusions can be drawn from the calculations and the tests. When calculations are performed on crossflow heat exchangers working with gas-to-gas the longitudinal heat conduction has normally such a great influence that it has to be taken into account. The influence can easily be obtained by means of the results given in this report. The conduction always lowers the effectiveness of the heat exchanger, and sometimes a better effectiveness can be obtained if the plate between the two airstreams has lower conductivity.

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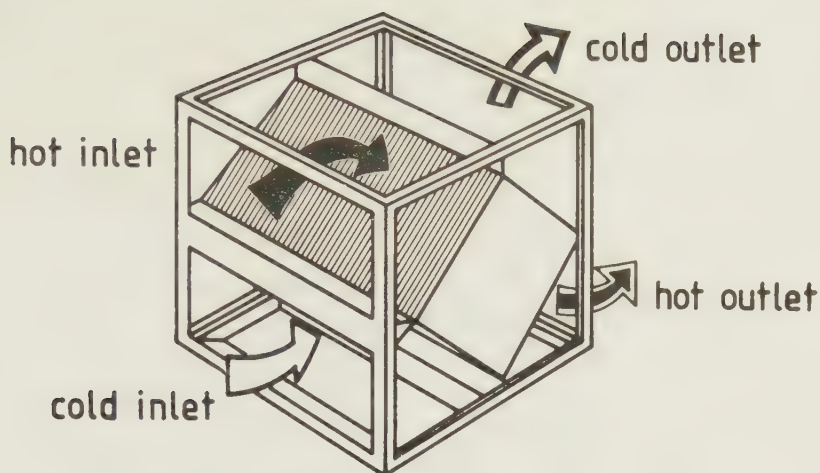


Fig 1a. Single-pass crossflow heat exchanger

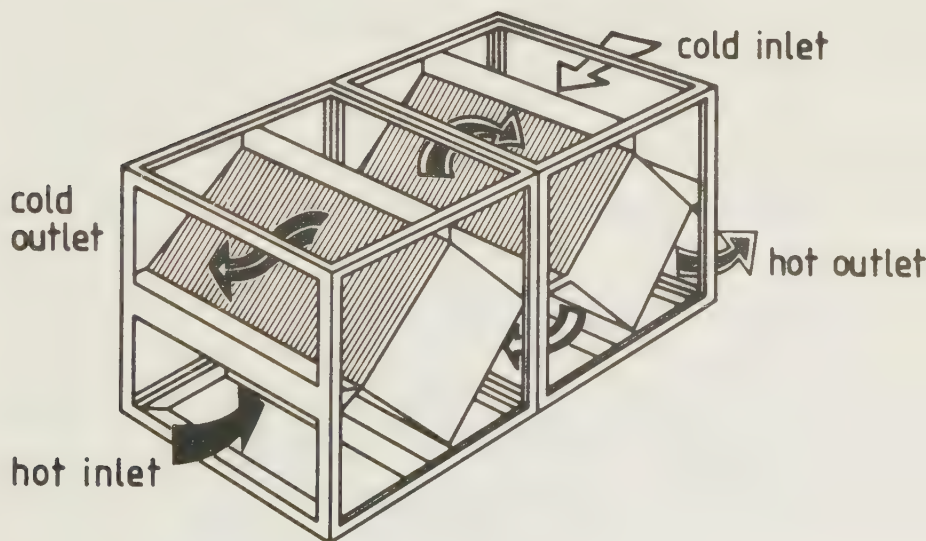


Fig 1b. Two-pass countercrossflow heat exchanger

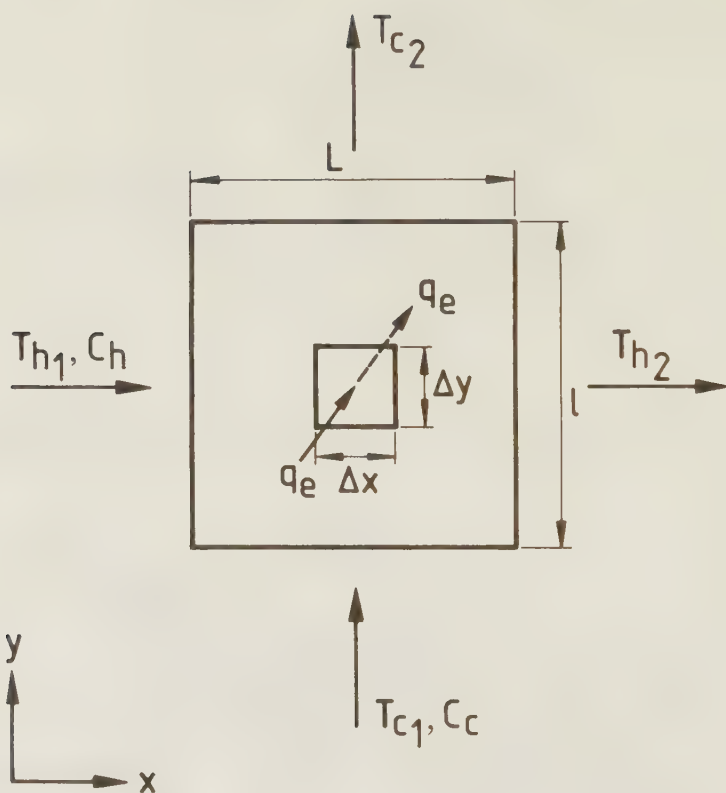


Fig 2a. Definition of capacities and temperatures.

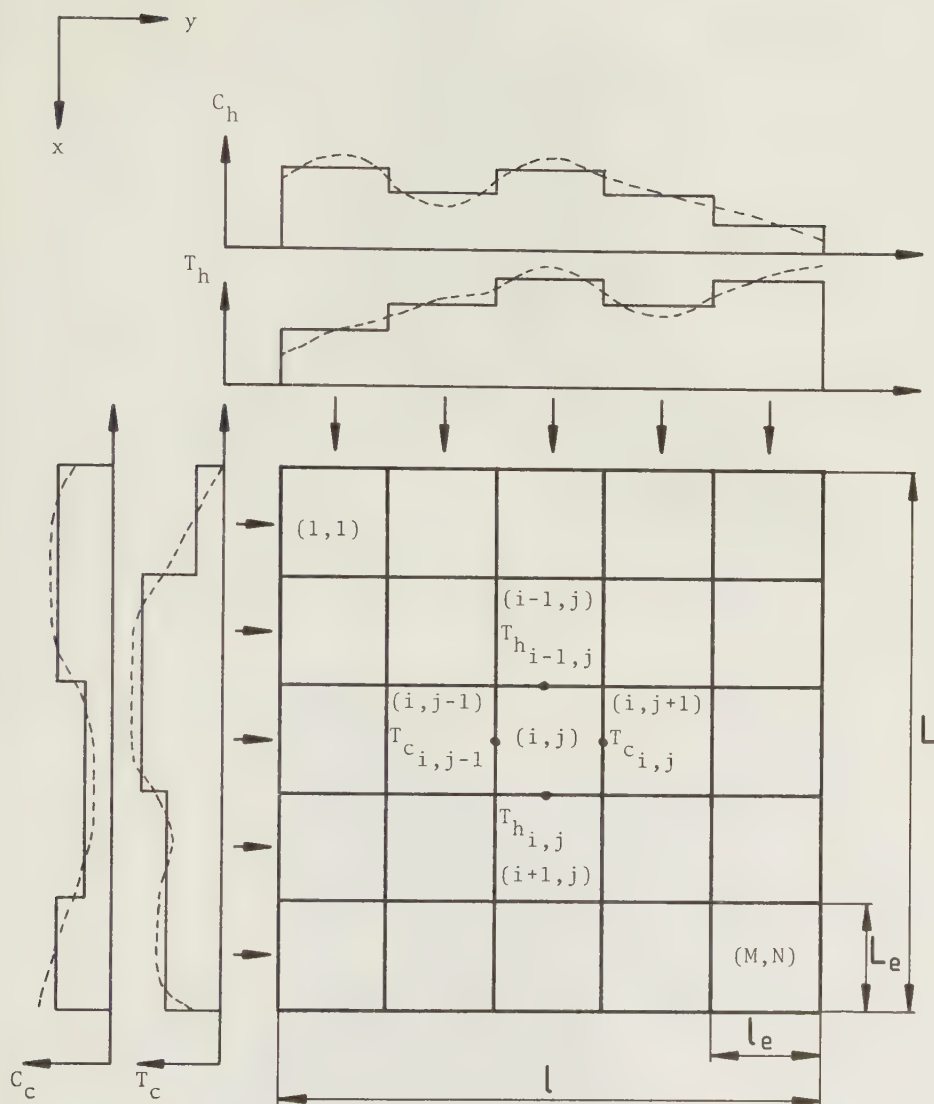


Fig 2b. Crossflow heat exchanger with arbitrary inlet mass velocities and temperatures.

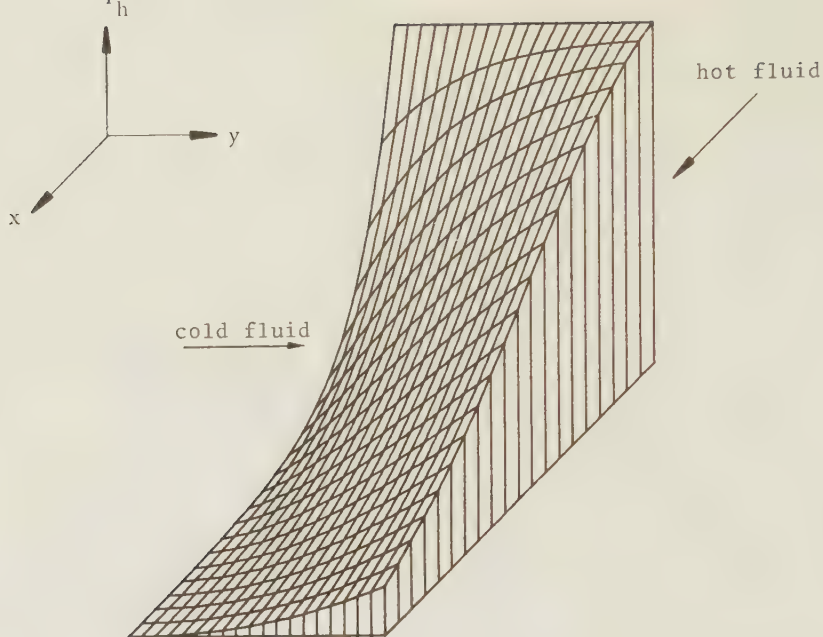


Fig 3. Hot side. $NTU_c = 4.0$, $NTU_h = 8.0$, uniform inlet temperatures and mass velocities, 20×20 elements.

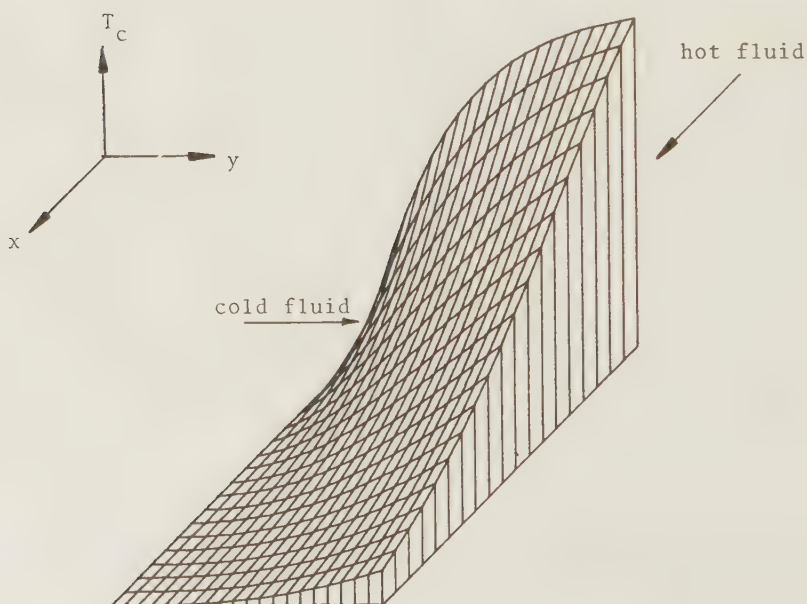


Fig 4. Cold side. $NTU_c = 4.0$, $NTU_h = 8.0$, uniform inlet temperatures and mass velocities, 20×20 elements.

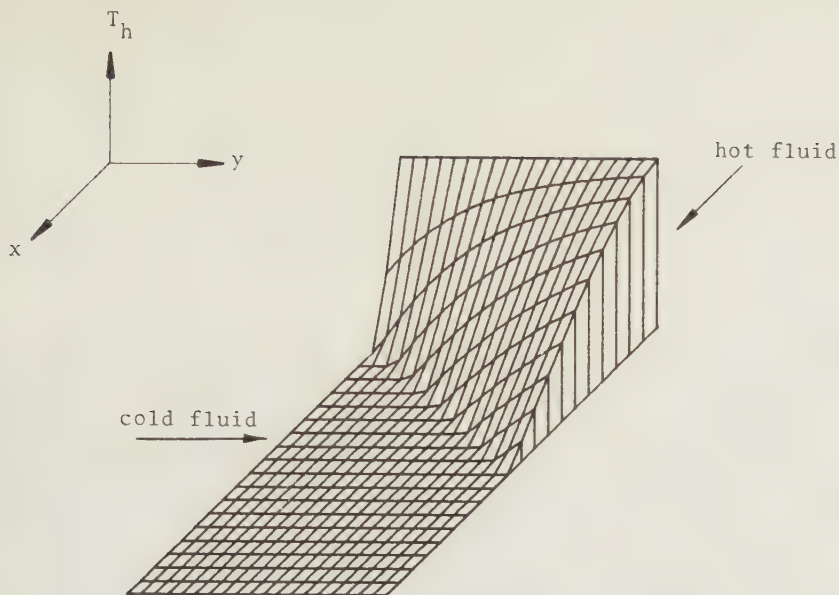


Fig 5. Hot side. $NTU_c = 4.0$, $NTU_h = 8.0$, uniform inlet temperatures and mass velocities, 20×20 elements, isotherm at 5000 (0.5).

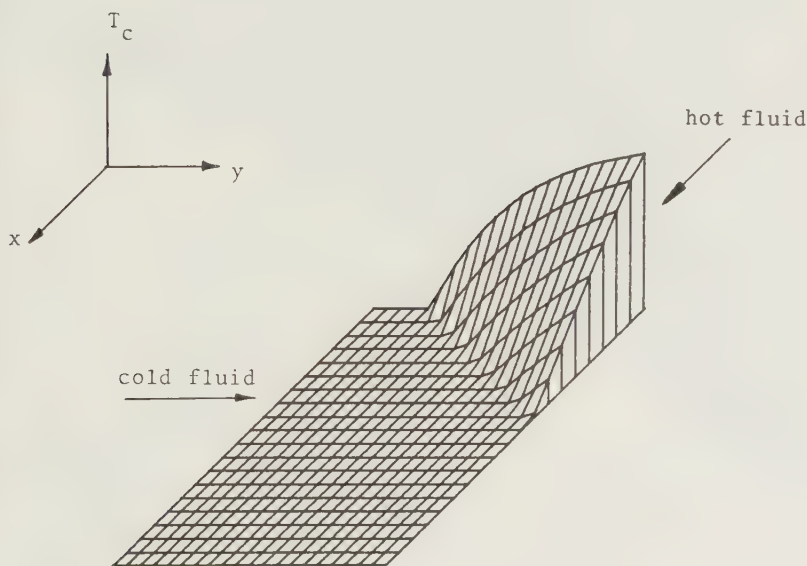


Fig 6. Cold side. $NTU_c = 4.0$, $NTU_h = 8.0$, uniform inlet temperatures and mass velocities, 20×20 elements, isotherm at 5000 (0.5).

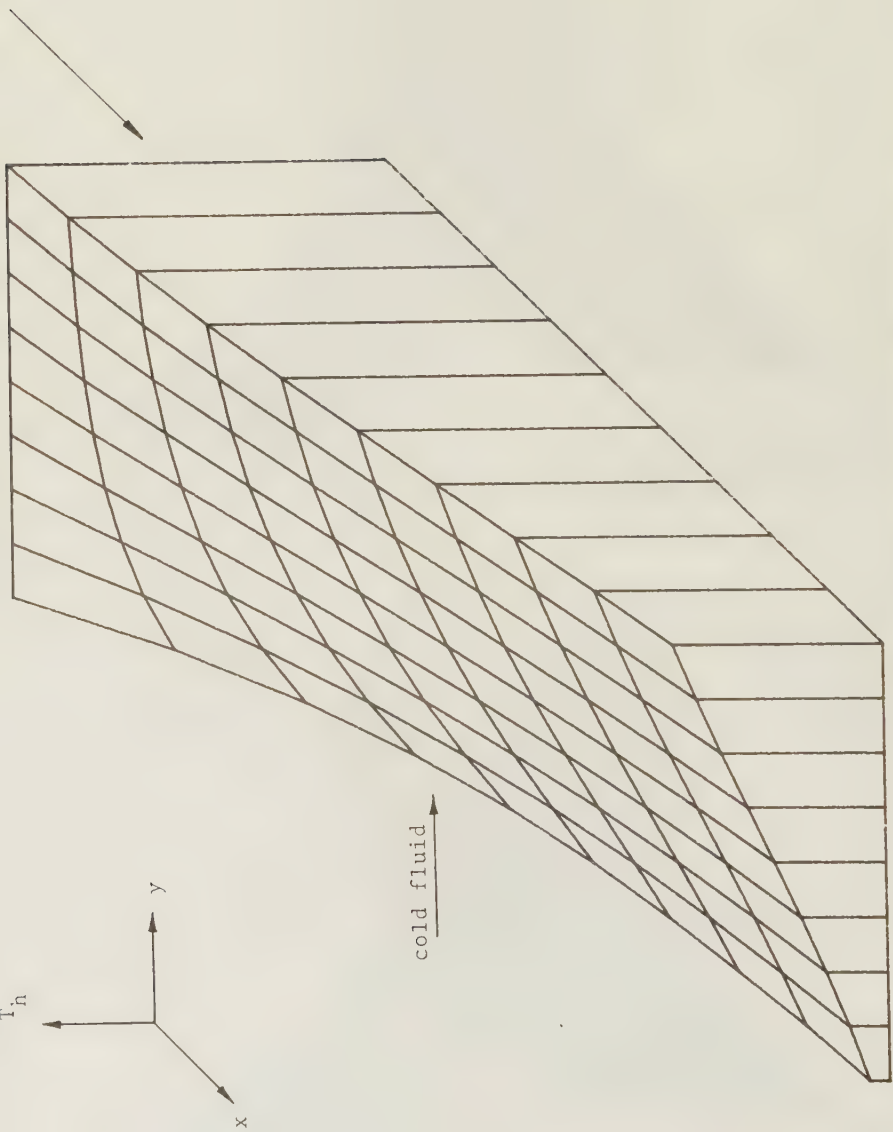


Fig 7. Hot side. $NTU_c = 3.6$, $NTU_h = 3.6$, uniform inlet temperatures and mass velocities, 9×9 elements, $NTU_{he\ i,j} = NTU_{ce\ i,j} = 0.4$.

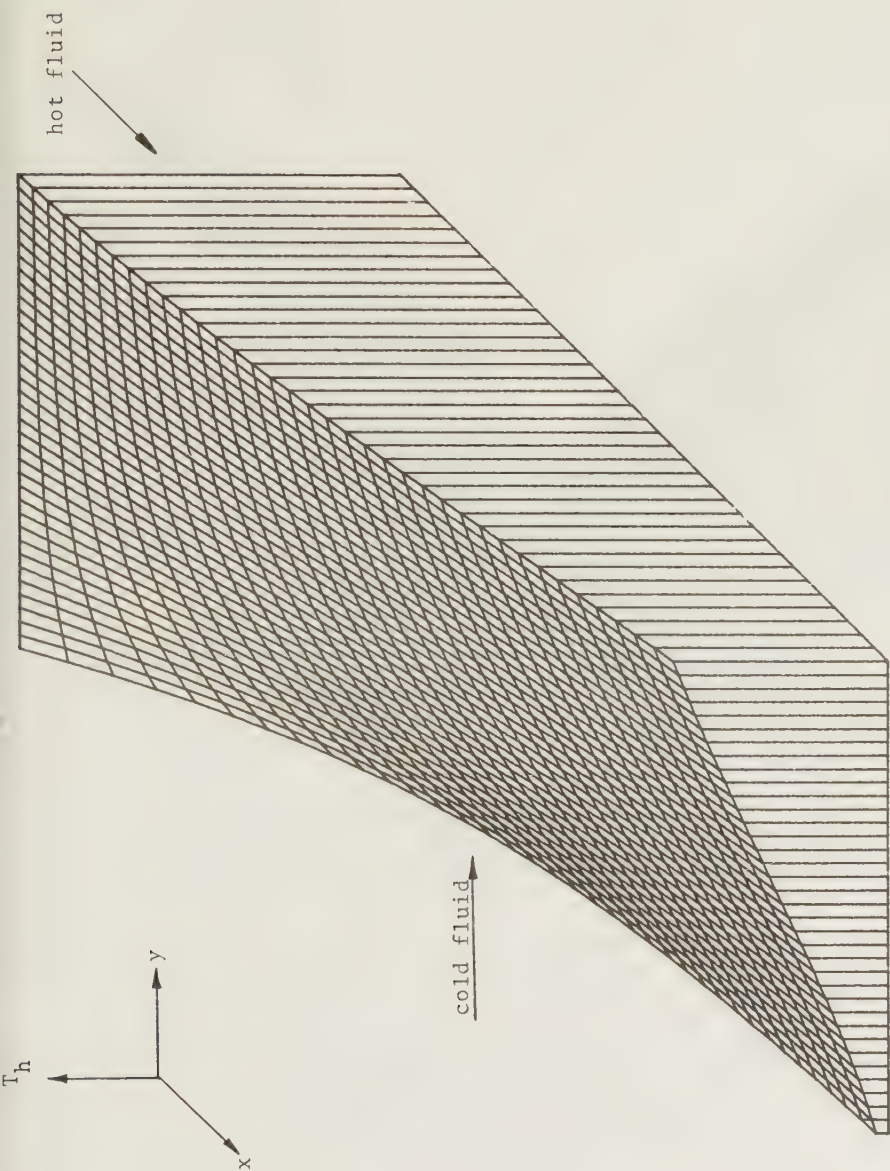


Fig 8. Hot side. $NTU_h = 3.6$, $NTU_c = 3.6$, uniform inlet temperatures and mass velocities, 36×36 elements, $NTU_{he,i,j} = NTU_{ce,i,j} = 0.1$.

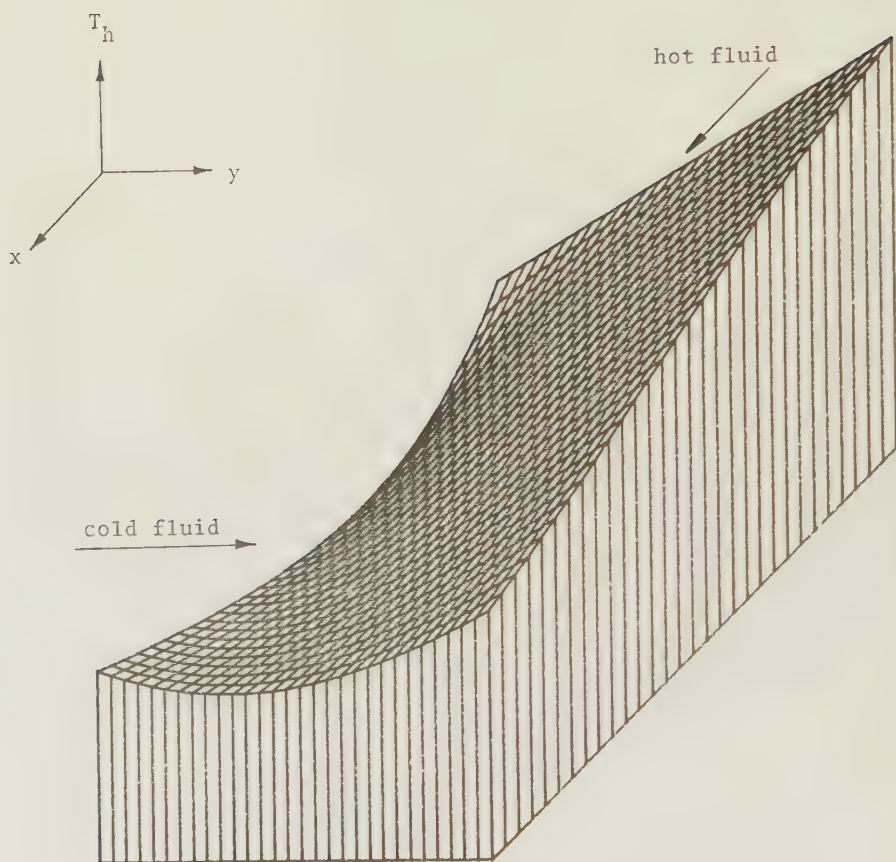


Fig 9. Hot side. $\dot{m}_{ce_{i,0}} = 0.02 \cdot i + 0.4$,

$$\dot{m}_{he_{0,j}} = 0.01 \cdot j + 0.7, T_{c_{i,0}} = (0.02 \cdot i) \cdot 10000,$$

$$T_{h_{0,j}} = (0.02 \cdot j + 0.4) \cdot 10000, \frac{A_{i,j} \cdot U}{C_p} = 0.1,$$

30x30 elements.

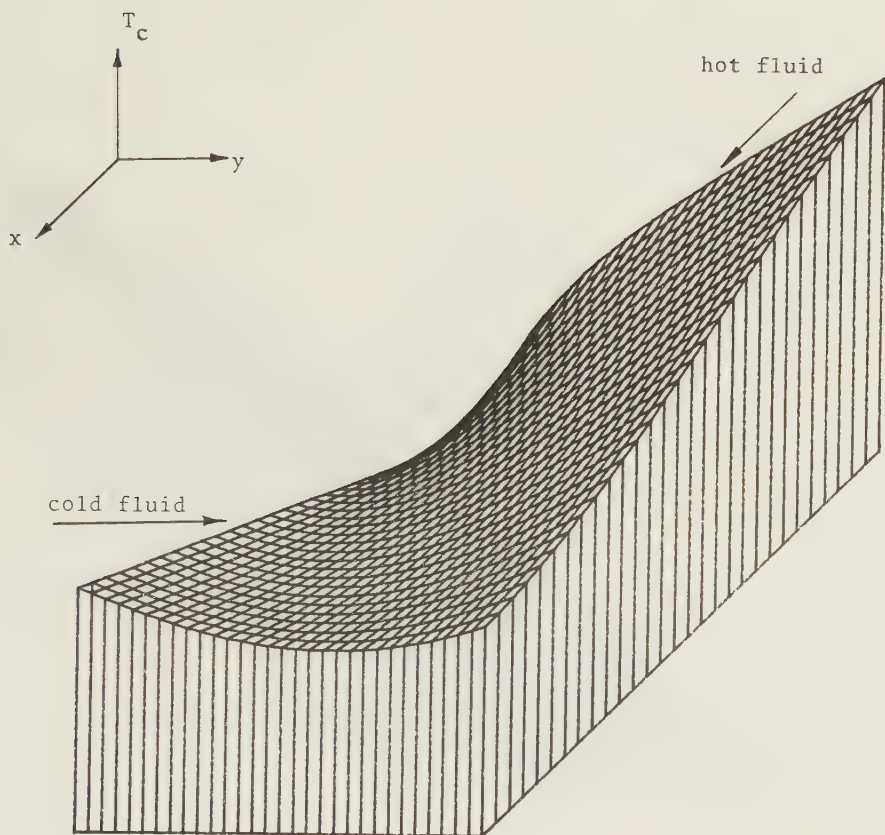


Fig 10. Cold side. $\dot{m}_{ce_{i,0}} = 0.02 i + 0.4$,

$$\dot{m}_{he_{0,j}} = 0.01 \cdot j + 0.7, T_{c_{i,0}} = (0.02 \cdot i) \cdot 10000,$$

$$T_{h_{0,j}} = (0.02 \cdot j + 0.4) \cdot 10000, \frac{A_{i,j} \cdot U}{C_p} = 0.1,$$

30×30 elements.

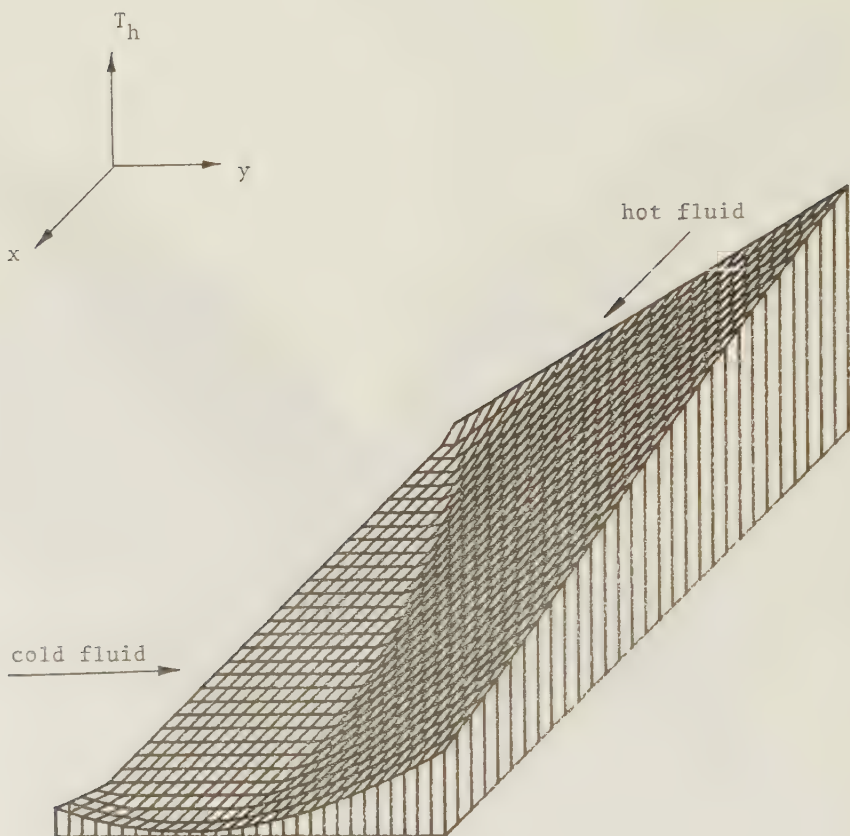


Fig 11. Hot side. $\dot{m}_{ce_{i,0}} = 0.02 i + 0.4$,

$$\dot{m}_{he_{0,j}} = 0.01 \cdot j + 0.7, T_{c_{i,0}} = (0.02 \cdot i) \cdot 10000,$$

$$T_{h_{0,j}} = (0.02 \cdot j + 0.4) \cdot 10000, \frac{A_{i,j} \cdot U}{C_p} = 0.1,$$

30×30 elements, isotherm at 4000 (0.4).

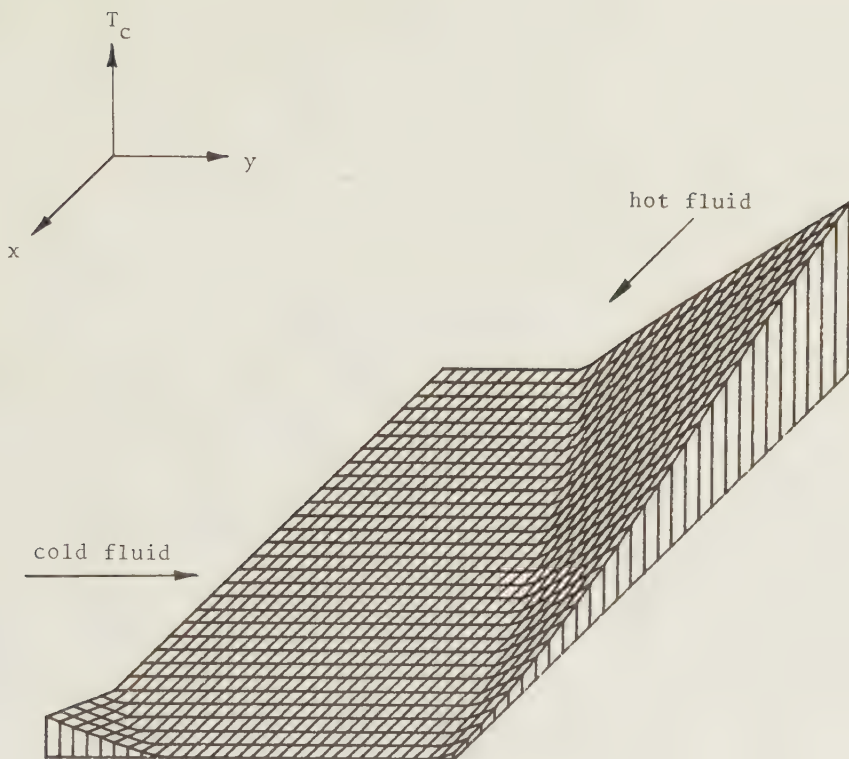


Fig 12. Cold side. $\dot{m}_{ce_{i,0}} = 0.02 i + 0.4$,
 $\dot{m}_{he_{0,j}} = 0.01 \cdot j + 0.7$, $T_{c_{i,0}} = (0.02 \cdot i) \cdot 10000$,
 $T_{h_{0,j}} = (0.02 \cdot j + 0.4) \cdot 10000$, $\frac{\Lambda_{i,j} \cdot U}{C_p} = 0.1$,
 30×30 elements, isotherm at 5000 (0.5).

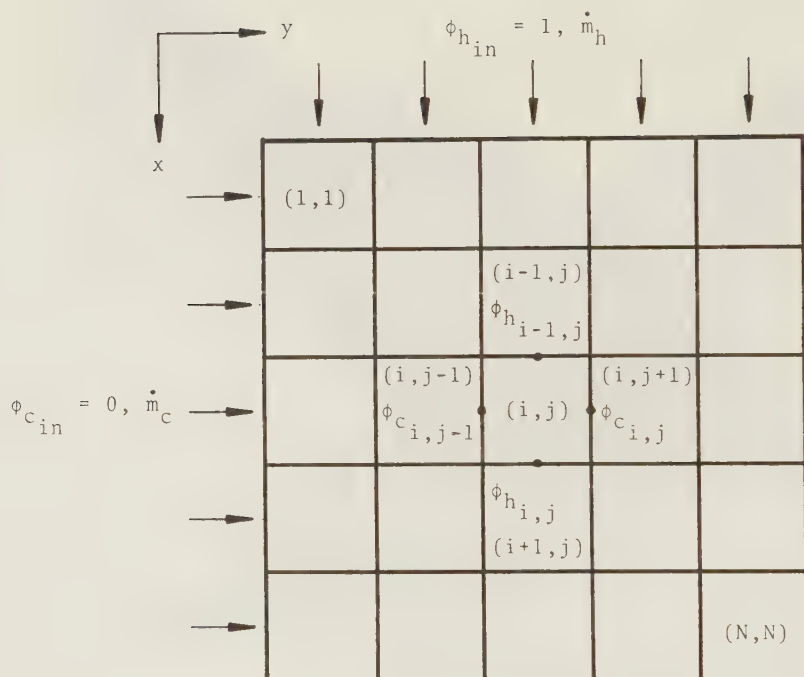


Fig 13. Crossflow heat exchanger divided into $N \times N$ elements.

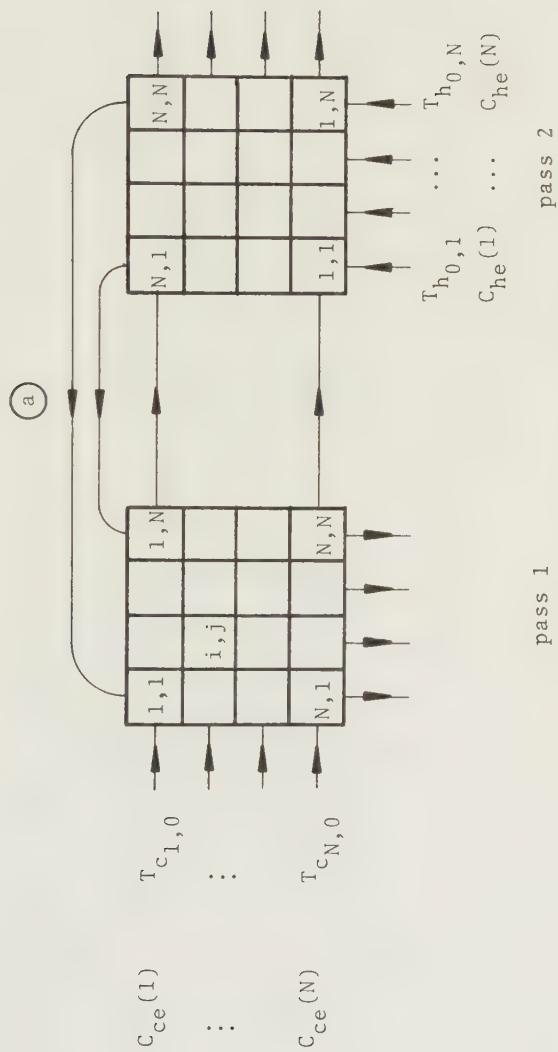


Fig 14. Two-pass arrangement.

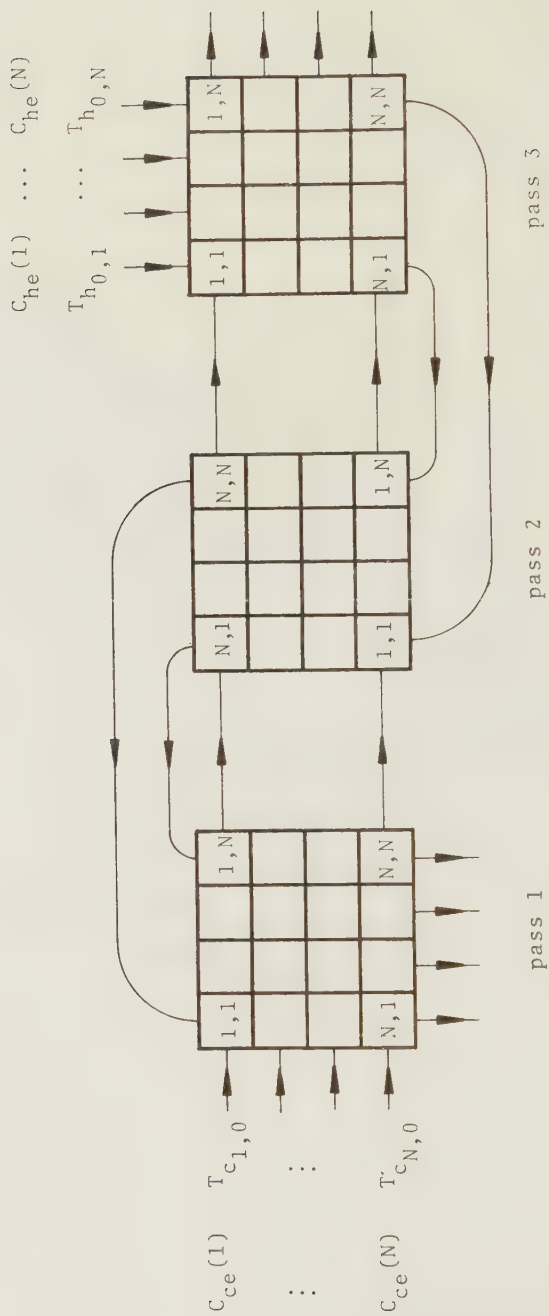


Fig 15. Three-pass arrangement.

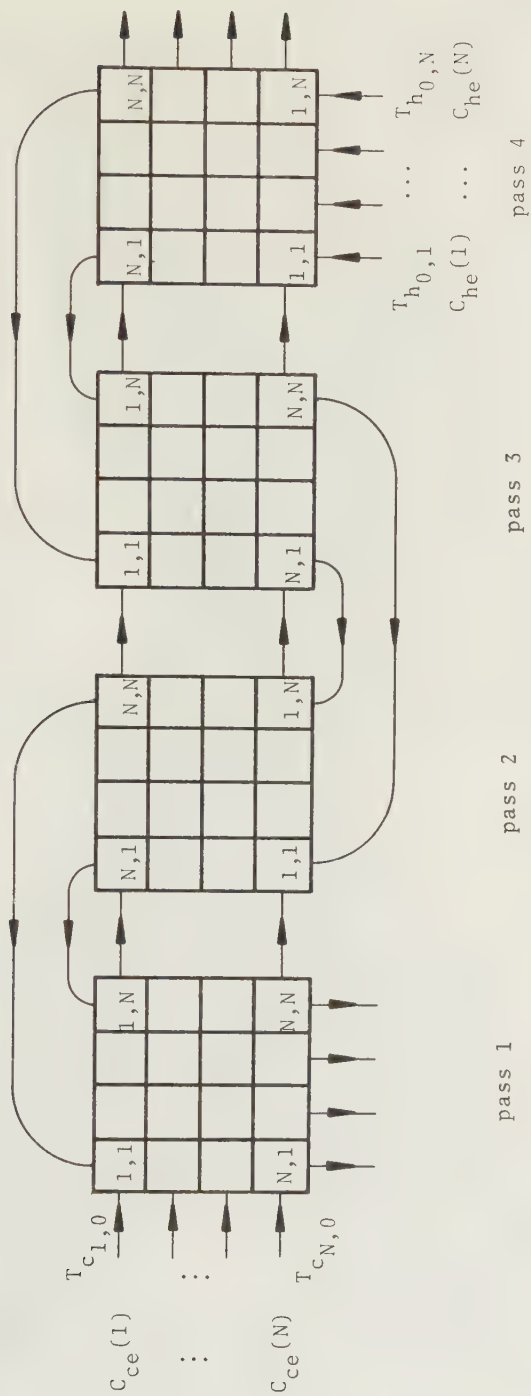


Fig 16. Four-pass arrangement.

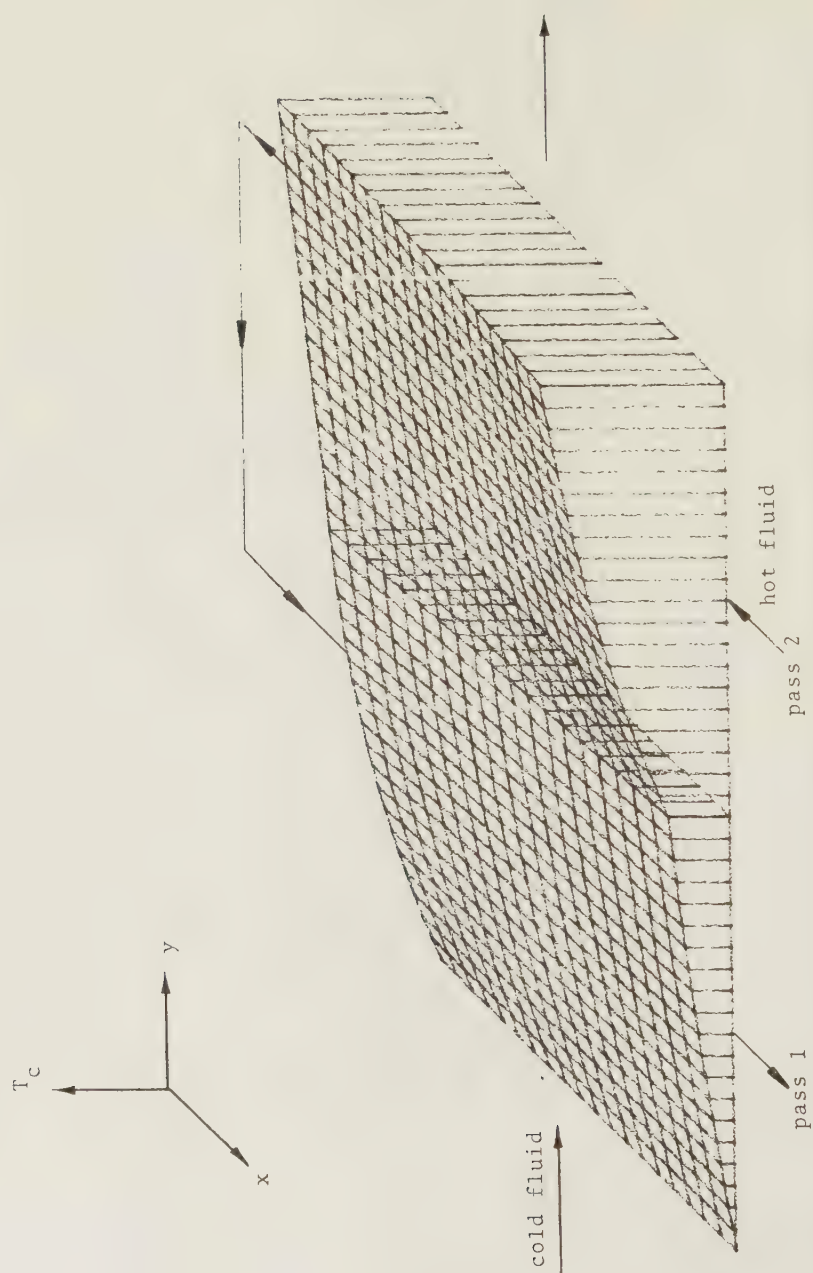


Fig 17. Cold side. 2-pass, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 elements in each pass.

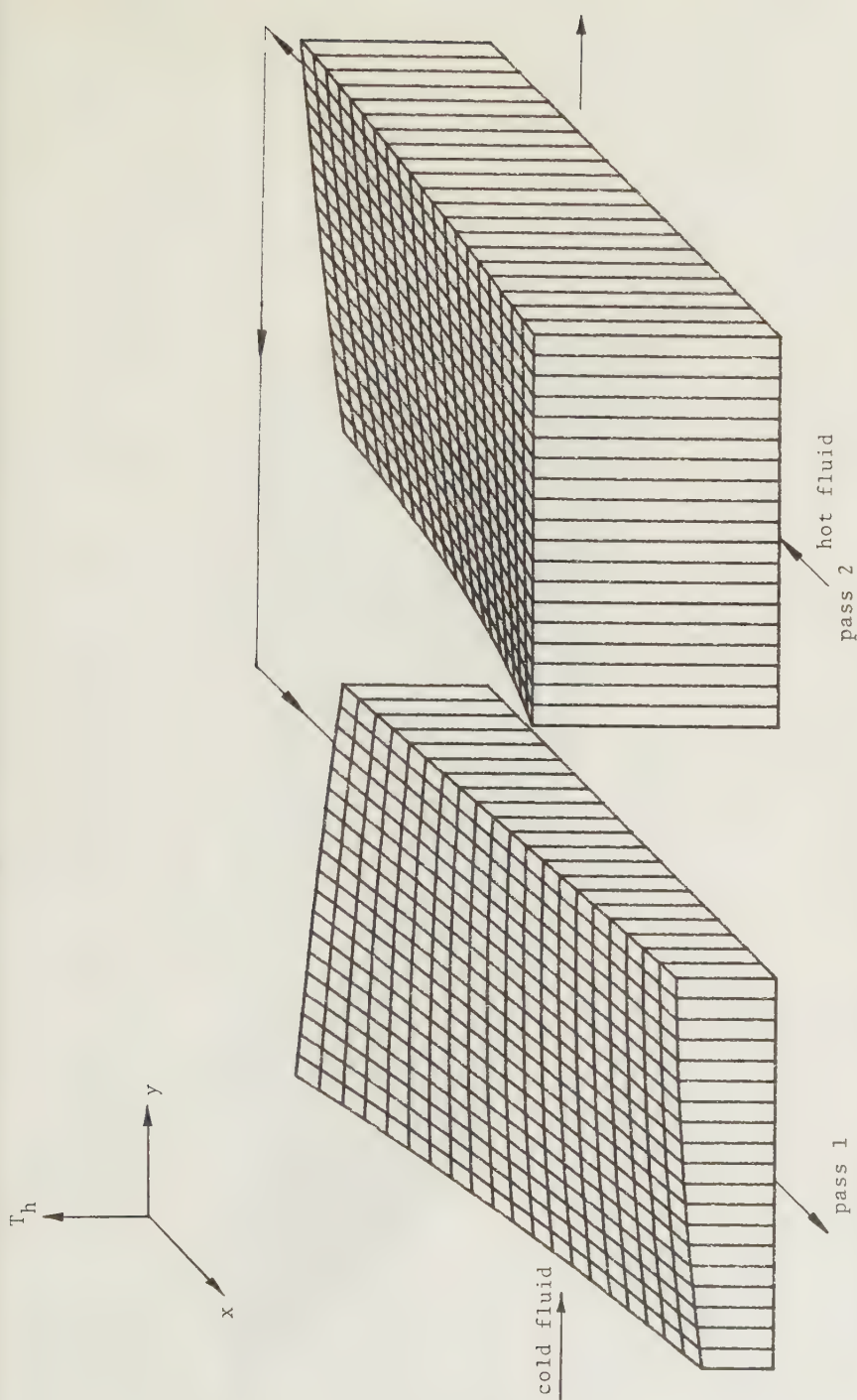


Fig 18. Hot side. 2-pass, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 elements in each pass.

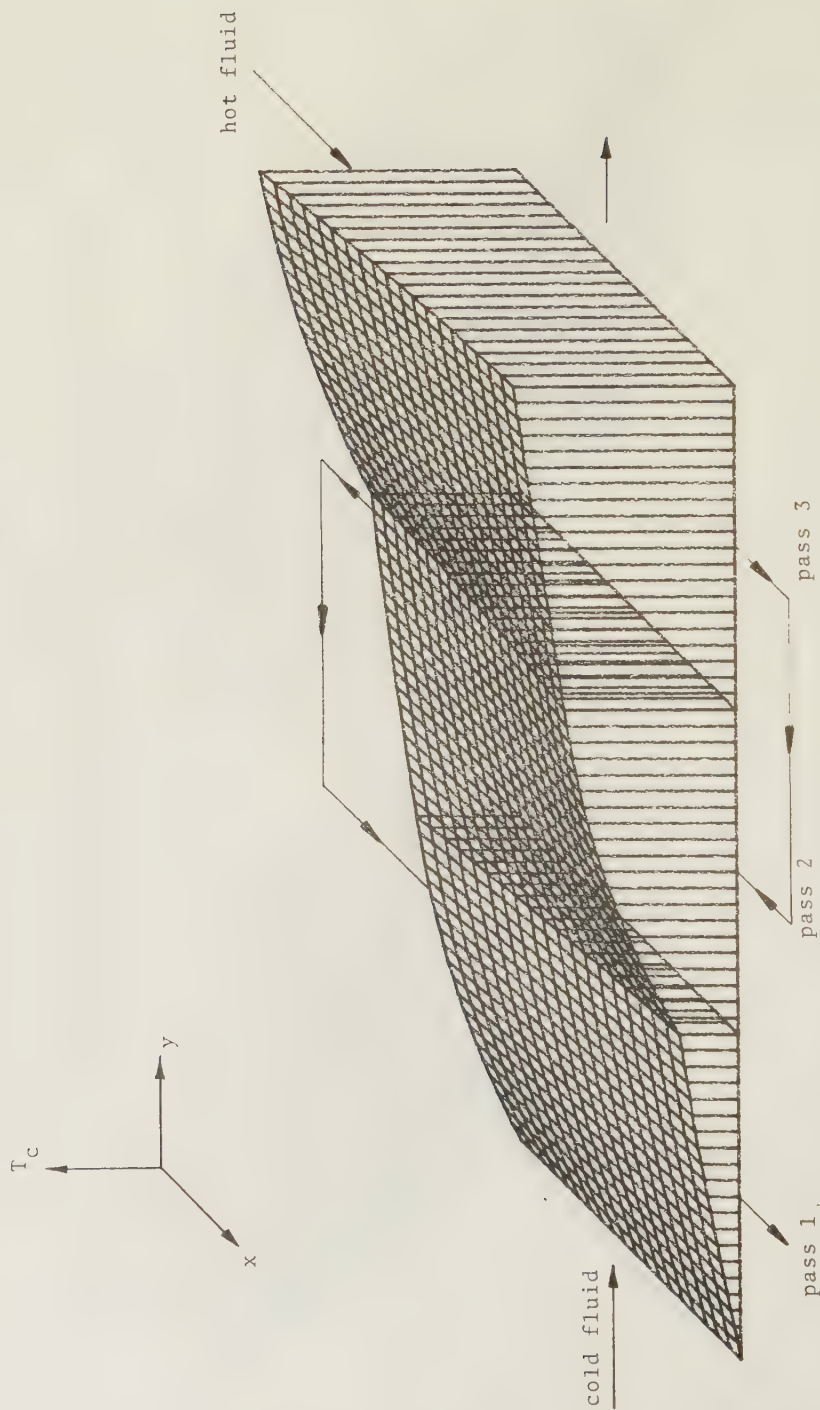


Fig 19. Cold side, 3-pass, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 elements in each pass.

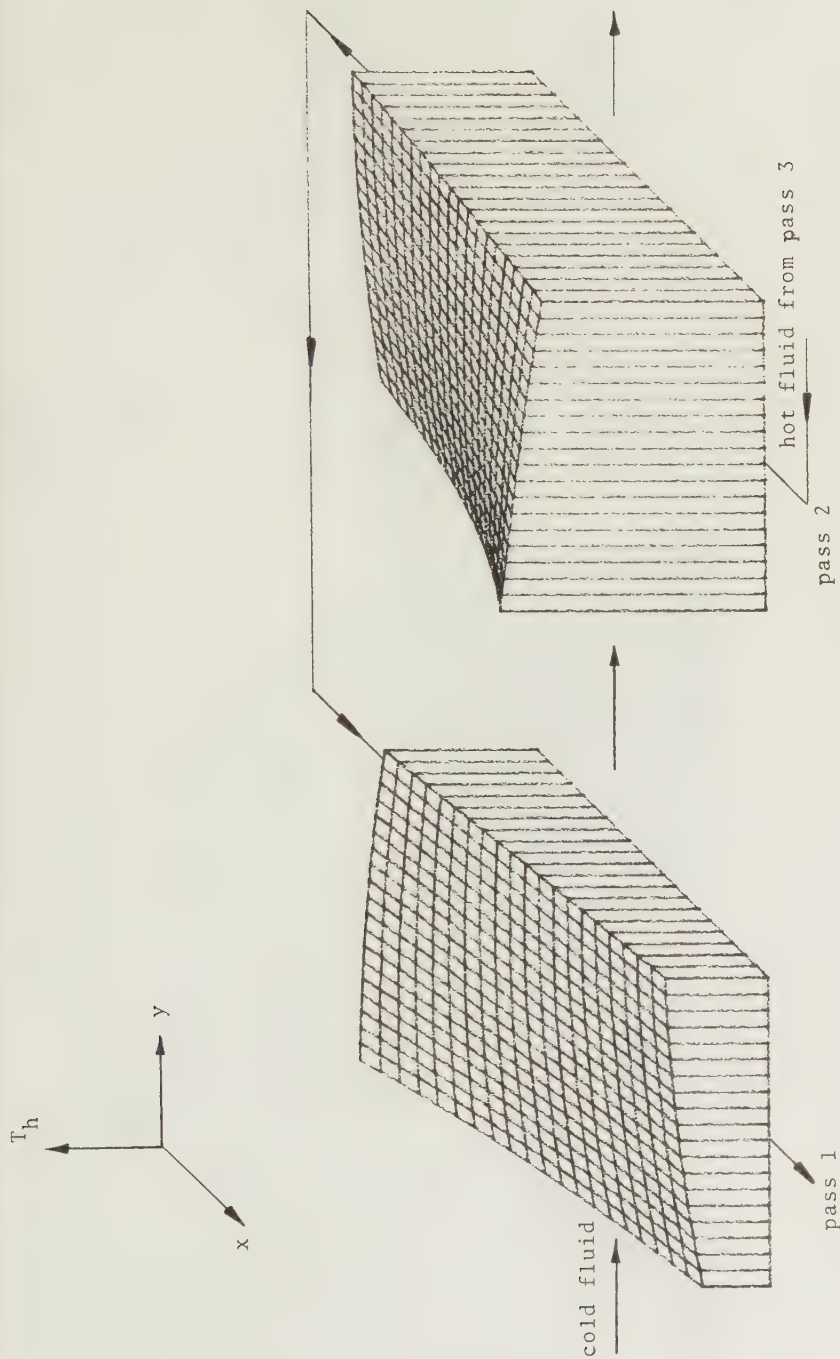


Fig 20. Hot side. 3-pass, pass 1 and 2, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 elements in each pass.

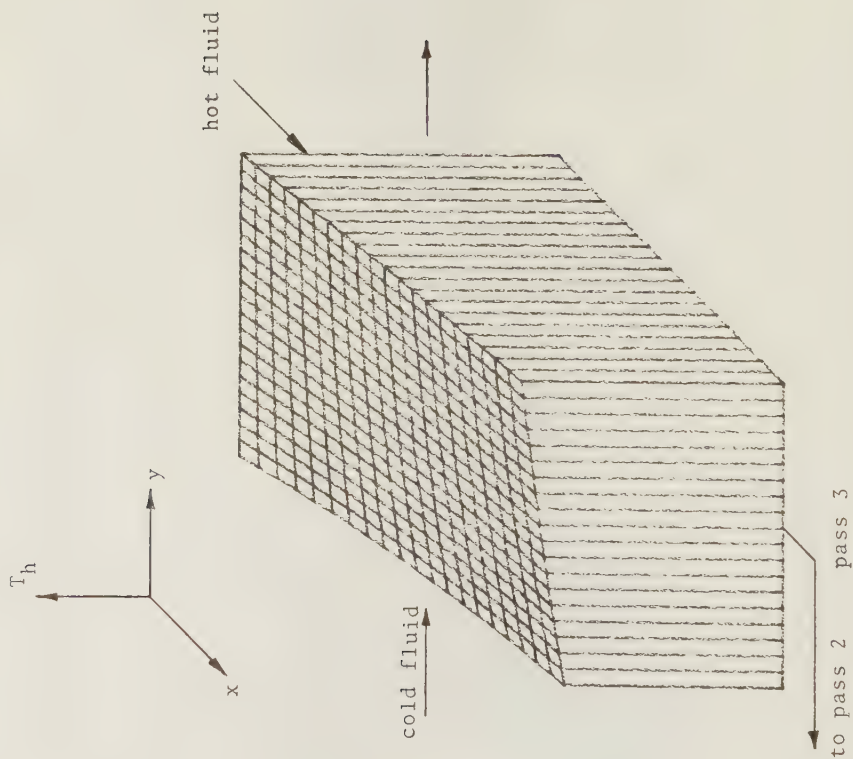


Fig 21. Hot side. 3-pass, pass 3, $NTU_c = NTU_h = 1.0$
for each pass, uniform inlet temperatures and
mass velocities, 20×20 elements in each pass.

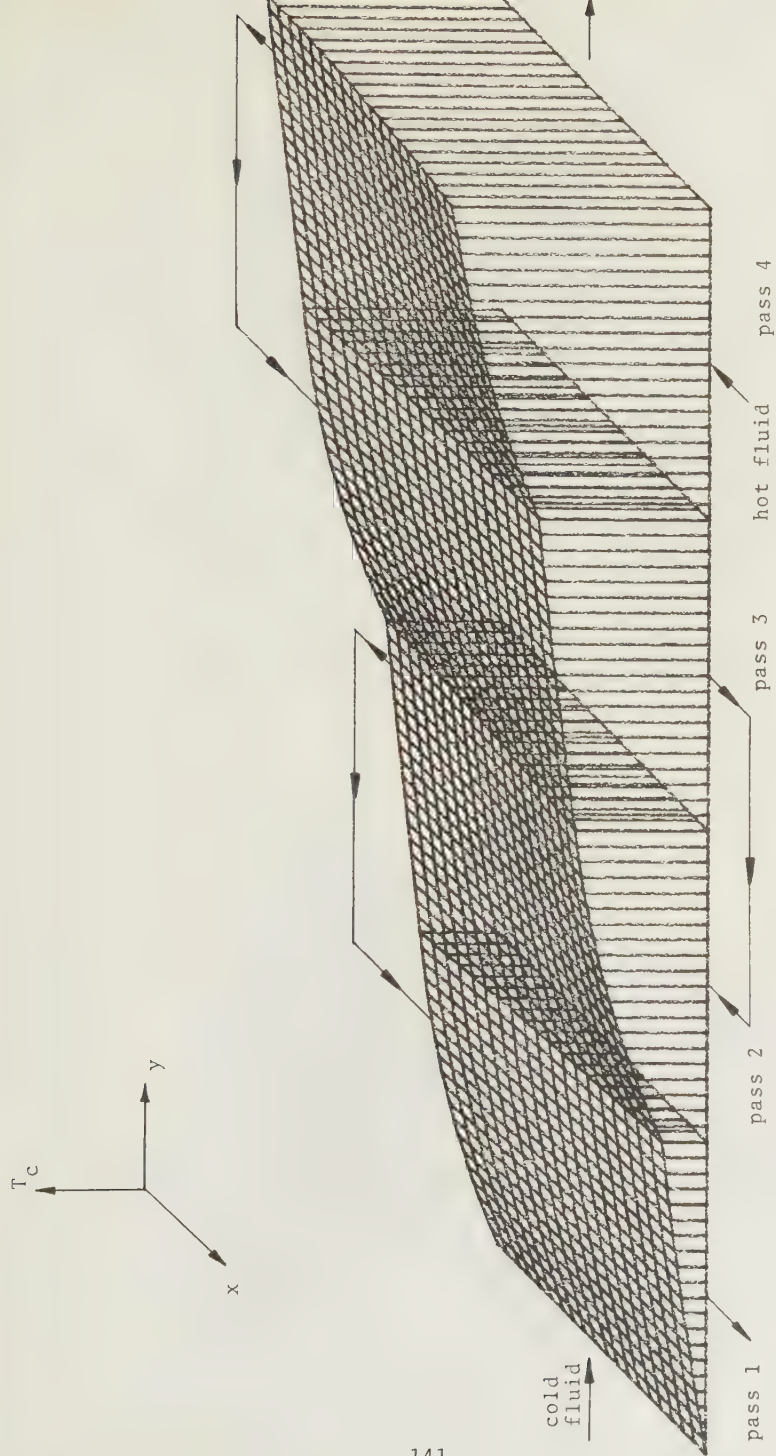


Fig 22. Cold side. 4-pass, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 elements in each pass.

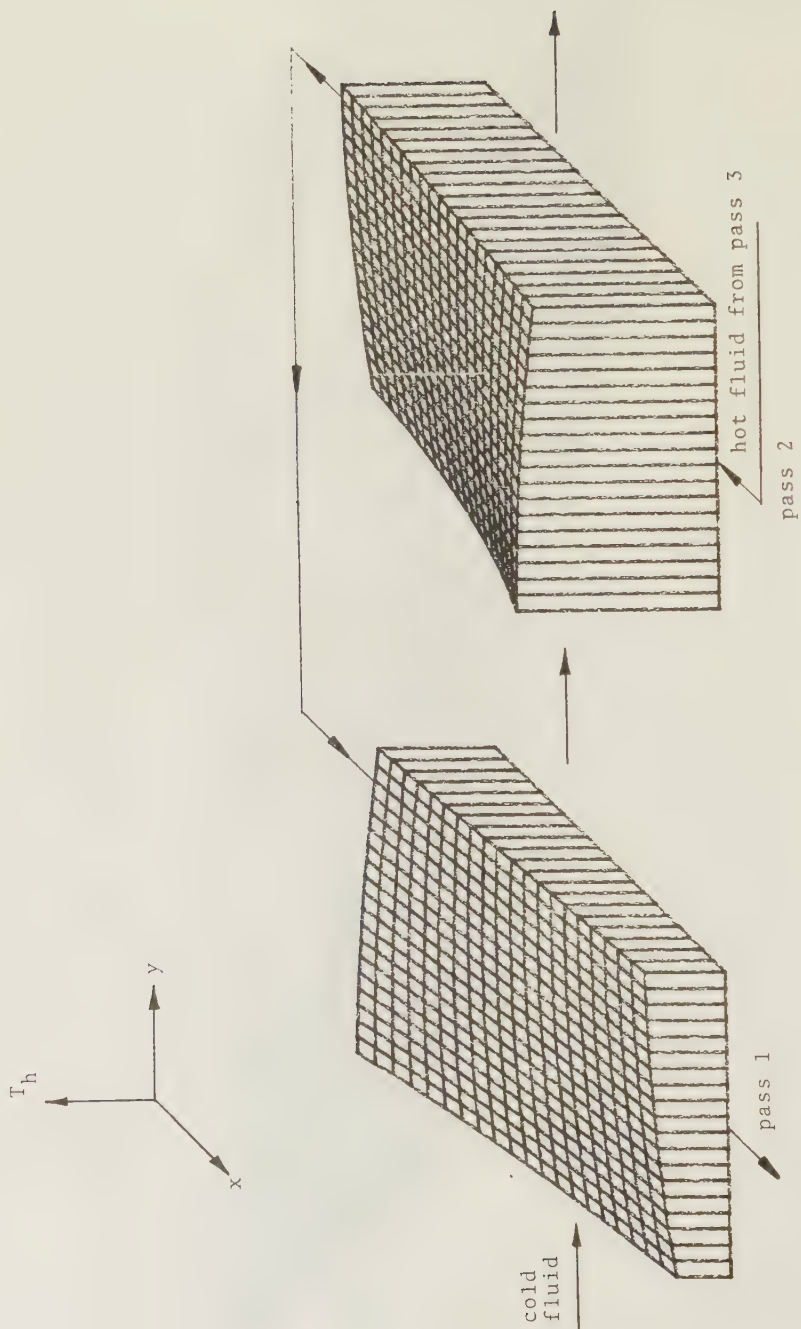


Fig 23. Hot side. 4-pass, pass 1 and 2, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 elements in each pass.

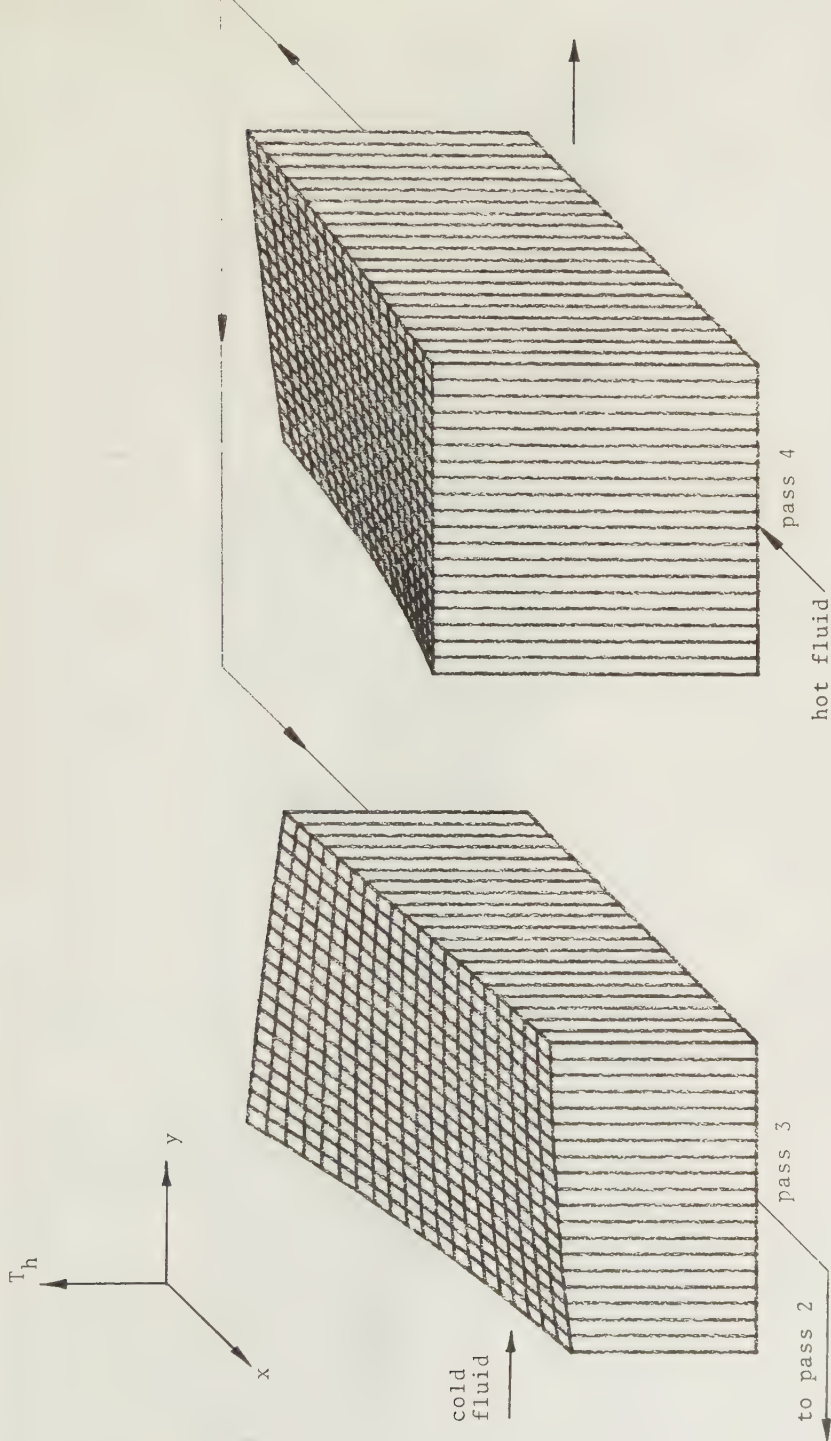


Fig 24. Hot side. 4-pass, pass 3 and 4, $NTU_c = NTU_h = 1.0$ for each pass, uniform inlet temperatures and mass velocities, 20×20 cells in each pass.

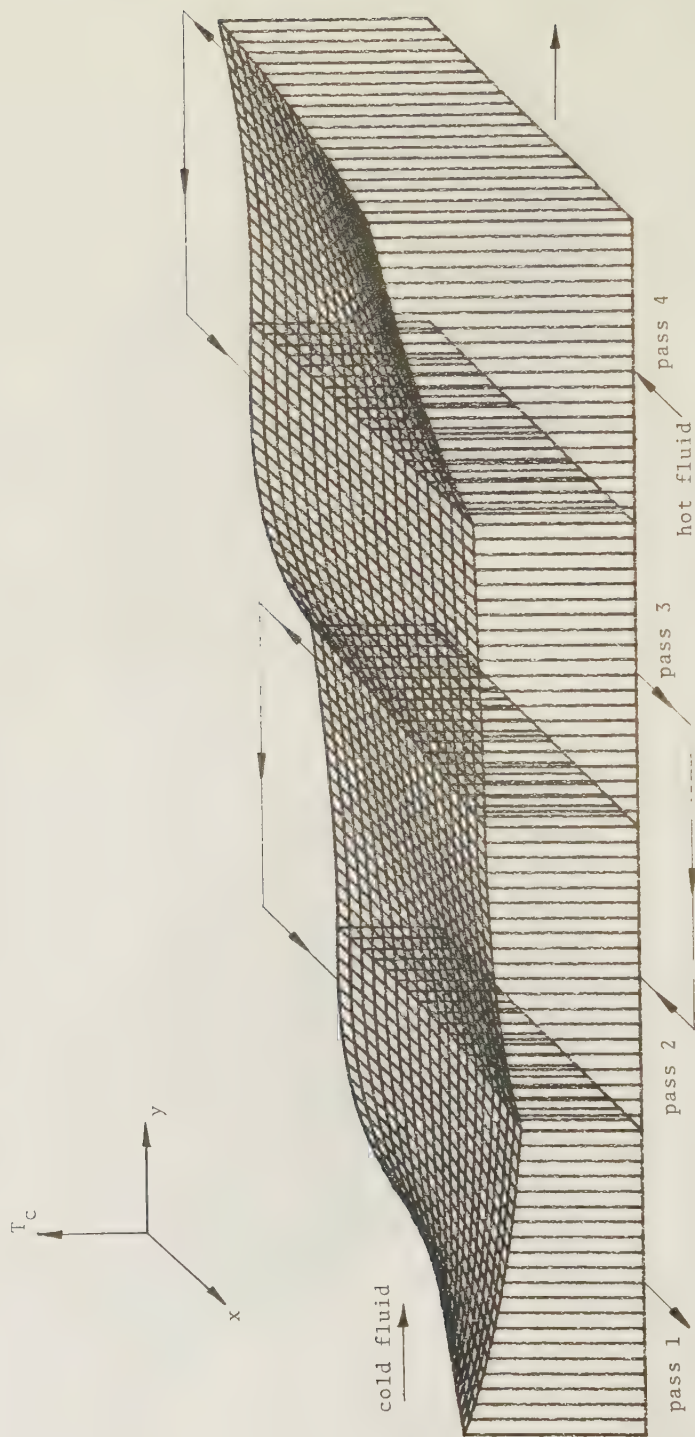


Fig 25. Cold side. 4-pass. $\dot{m}_{cei,0} = 0.8 + 0.01 \cdot i$, $\dot{m}_{hc_0,j} = 0.5 + 0.025 \cdot j$, $T_{ci,0} = 0.03 \cdot i \cdot 10000$,
 $T_{h0,j} = (0.8 + 0.01 \cdot j) \cdot 10000$, $\frac{U \cdot A_{i,j}}{C_p} = 0.1$, 20x20 elements in each pass.

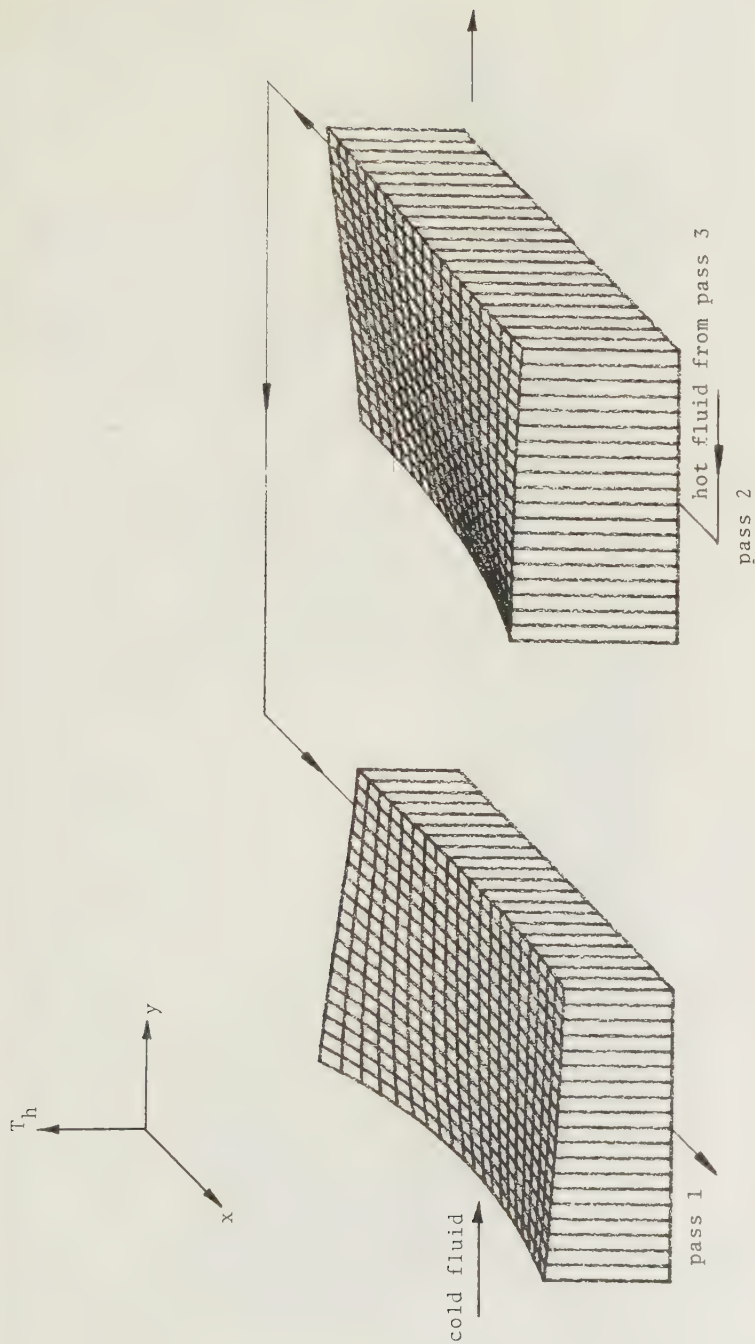


Fig 26. Hot side. 4-pass, pass 1 and 2, $\dot{m}_{ce_{i,0}} = 0.8 + 0.01 \cdot i$, $\dot{m}_{he_{0,j}} = 0.5 + 0.025 \cdot j$,

$$T_{c_{i,0}} = 0.03 \cdot i \cdot 10000, T_{h_{0,j}} = (0.8 + 0.01 \cdot j) \cdot 10000, \frac{U \cdot A_{i,j}}{C_p} = 0.1, 20 \times 20 \text{ elements in each pass.}$$

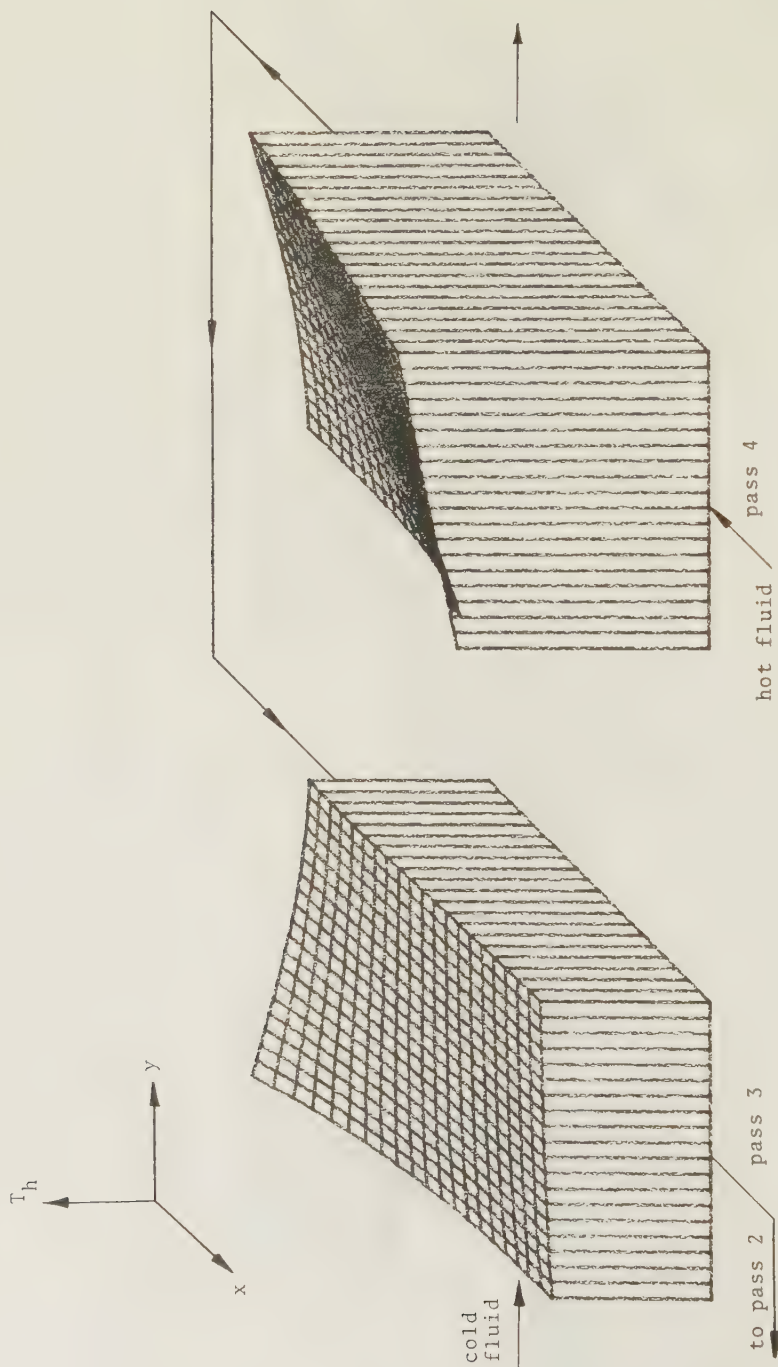


Fig 27. Hot side. 4-pass, pass 3 and 4, $\dot{m}_{ce,i,0} = 0.8 + 0.01 \cdot i$, $\dot{m}_{he,0,j} = 0.5 + 0.025 \cdot j$,

$$T_{ci,0} = 0.03 \cdot i \cdot 10000, T_{h0,j} = (0.8 + 0.01 \cdot j) \cdot 10000, \frac{U \cdot A_{i,j}}{C_p} = 0.1, 20 \times 20 \text{ elements in each pass.}$$

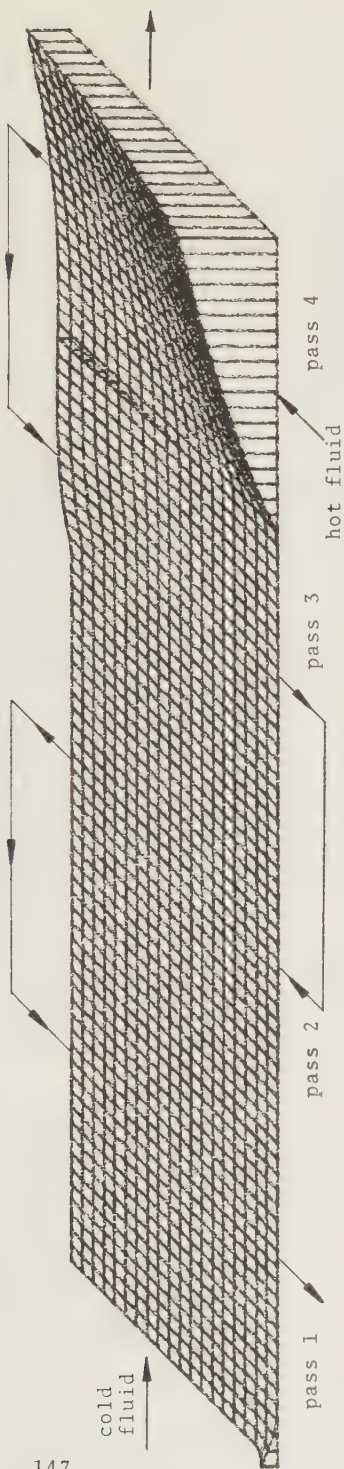
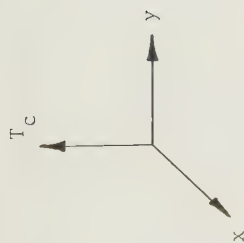


Fig 28. Cold side. 4-pass, $\dot{m}_{ce,i,0} = 0.8 + 0.01 \cdot i$, $\dot{m}_{he0,j} = 0.5 + 0.025 \cdot j$, $T_{c,i,0} = 0.03 \cdot i \cdot 10000$,

$$T_{h0,j} = (0.8 + 0.01 \cdot j) \cdot 10000, \quad \frac{U \cdot A_{i,j}}{C_p} = 0.1, \quad 20 \times 20 \text{ elements in each pass, isotherm at}$$

$$T_c = 5500.$$

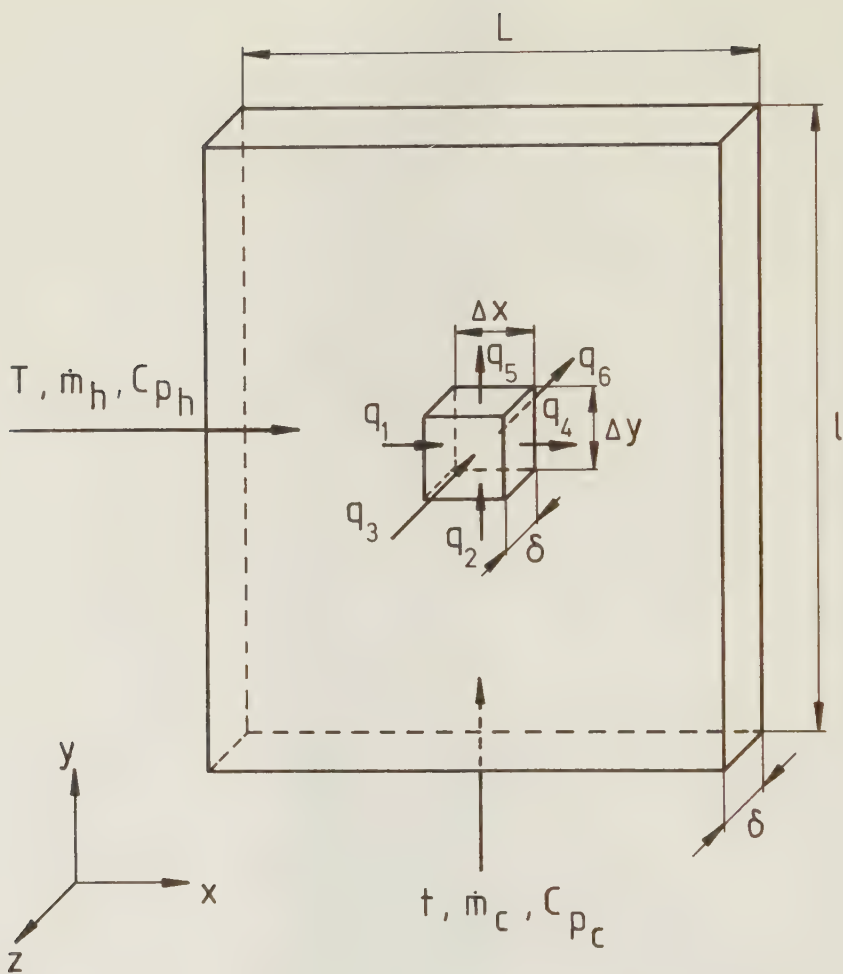
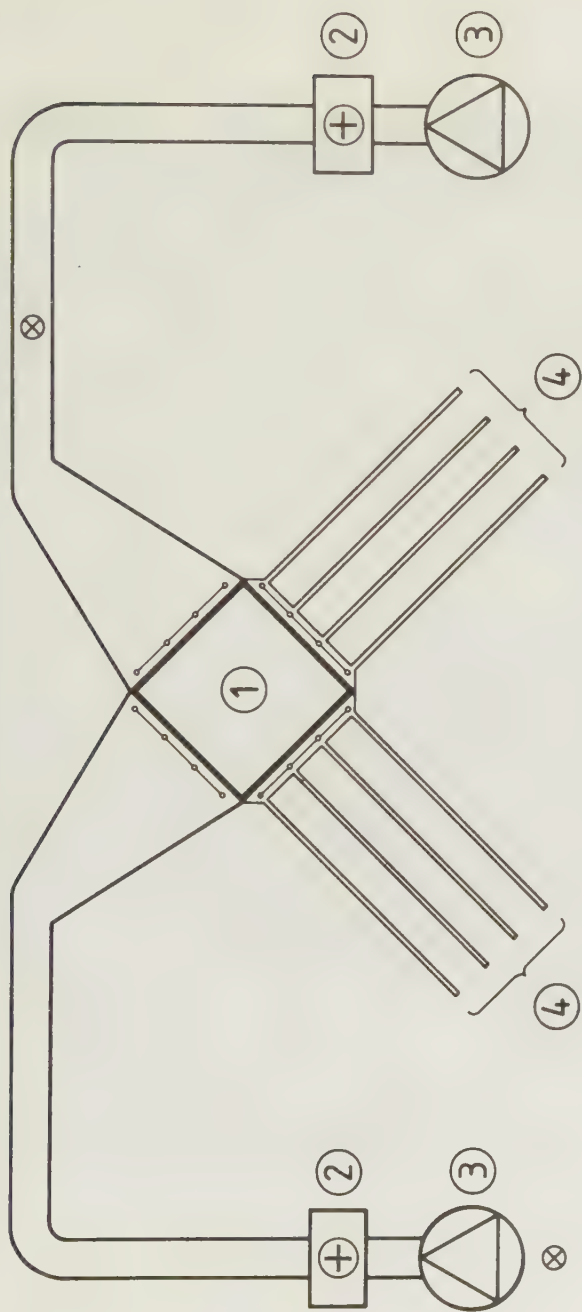


Fig 29. Heat transfer in the wall of a crossflow heat exchanger.



1. Heat exchanger
 2. Electric heater
 3. Fan
 4. Measurement tubes
- Thermocouples
 ⊗ Humidity probe

Fig 30. Test rig for a cross-flow heat exchanger.



Fig 31. Tested heat exchanger.
The numbers on the heat exchanger are
related to the measurement tubes.

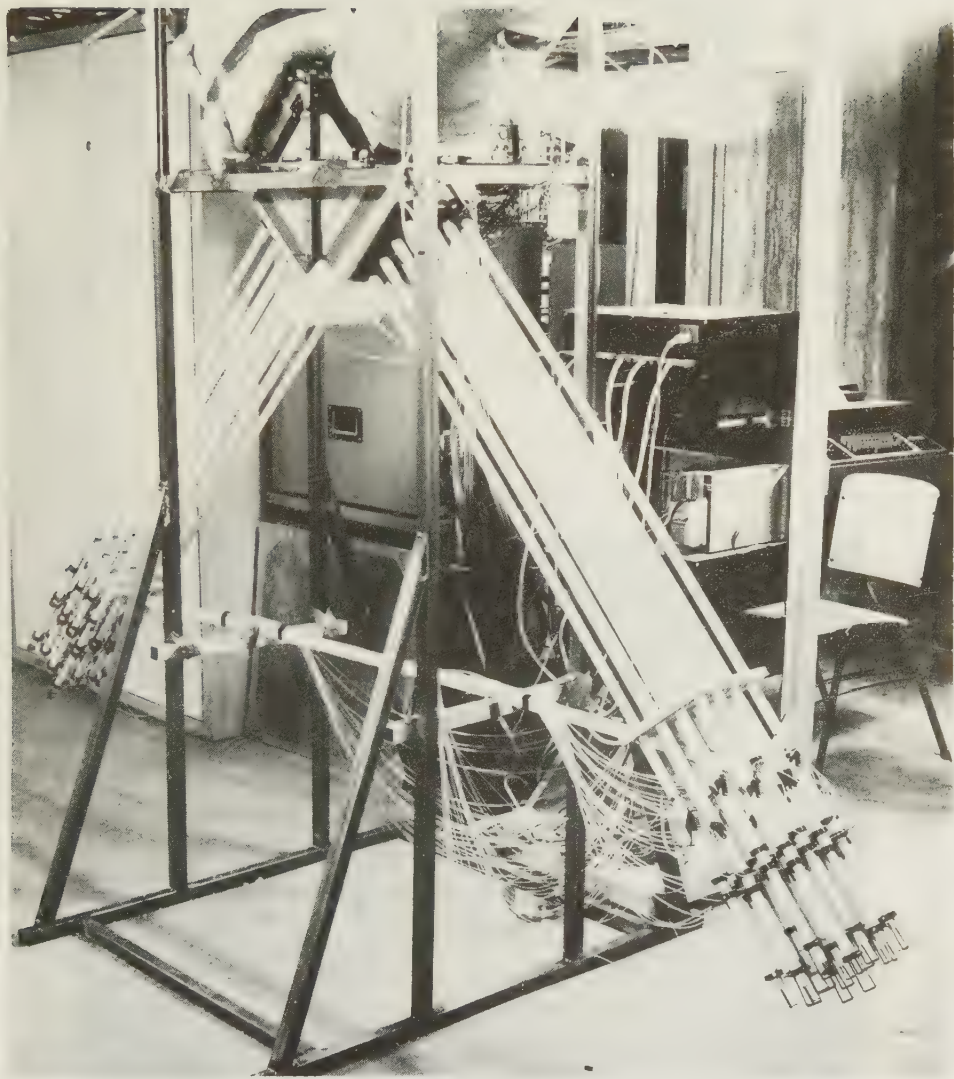


Fig 32a. Mass flow rate measurement tubes.

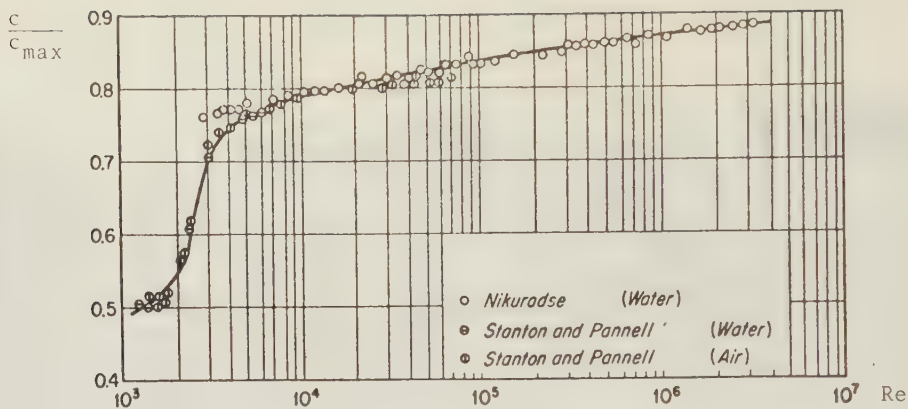


Fig 32b. $\frac{c}{c_{\max}}$ as a function of the Reynolds number [19, 20].

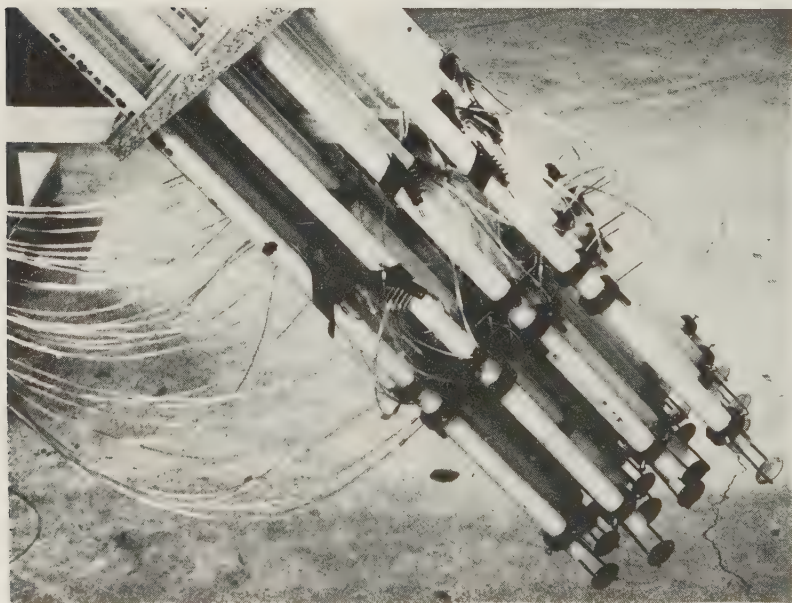


Fig 33. Adjustable plates to control the flow rate through the heat exchanger.

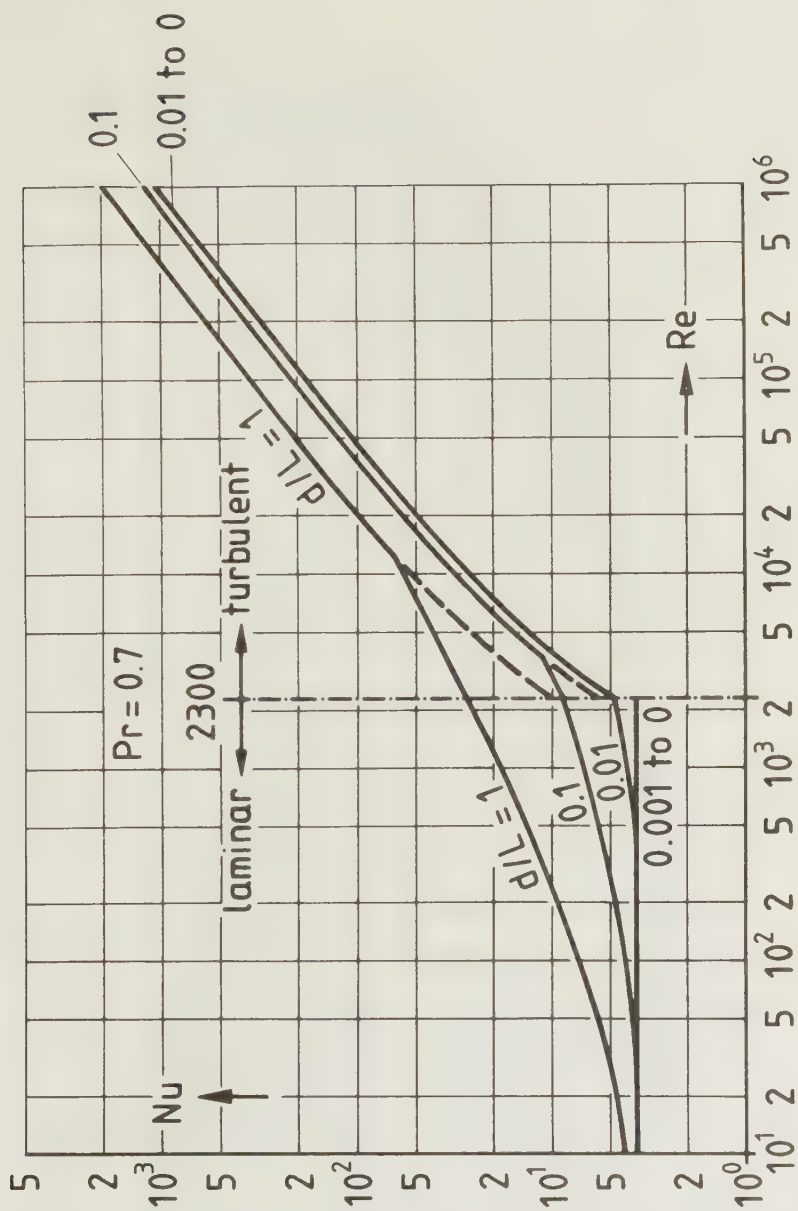


Fig 34. Heat transfer in tubes [23].

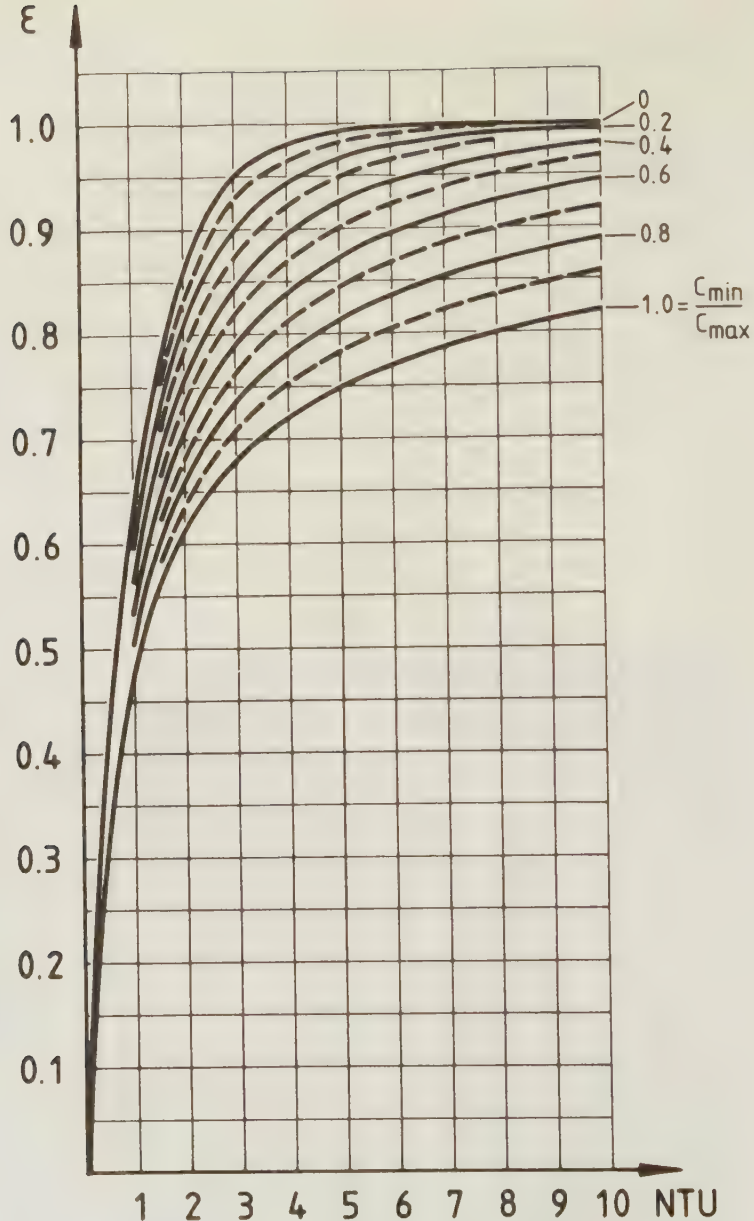


Fig 35. Effectiveness (ϵ) for a single pass crossflow heat exchanger without mixing and longitudinal heat conduction as a function of NTU and $\frac{C_{min}}{C_{max}}$.

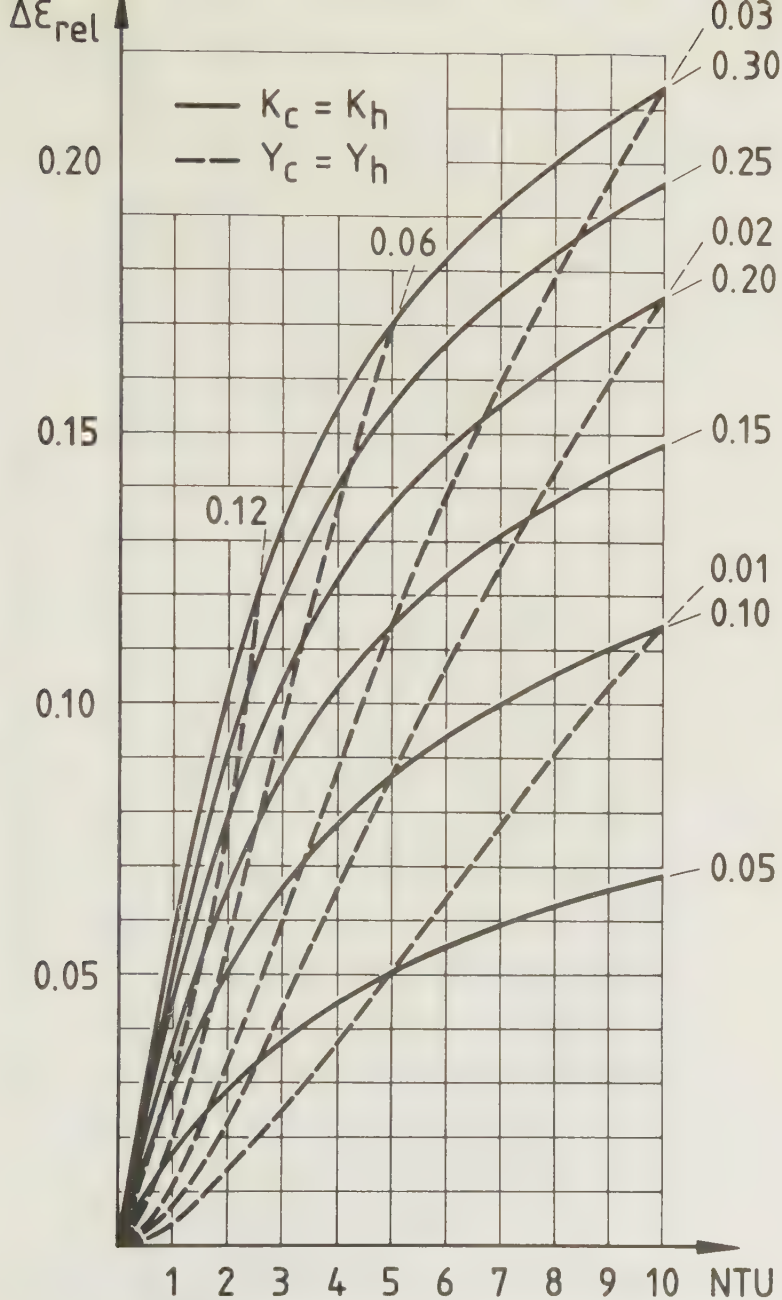


Fig 36. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

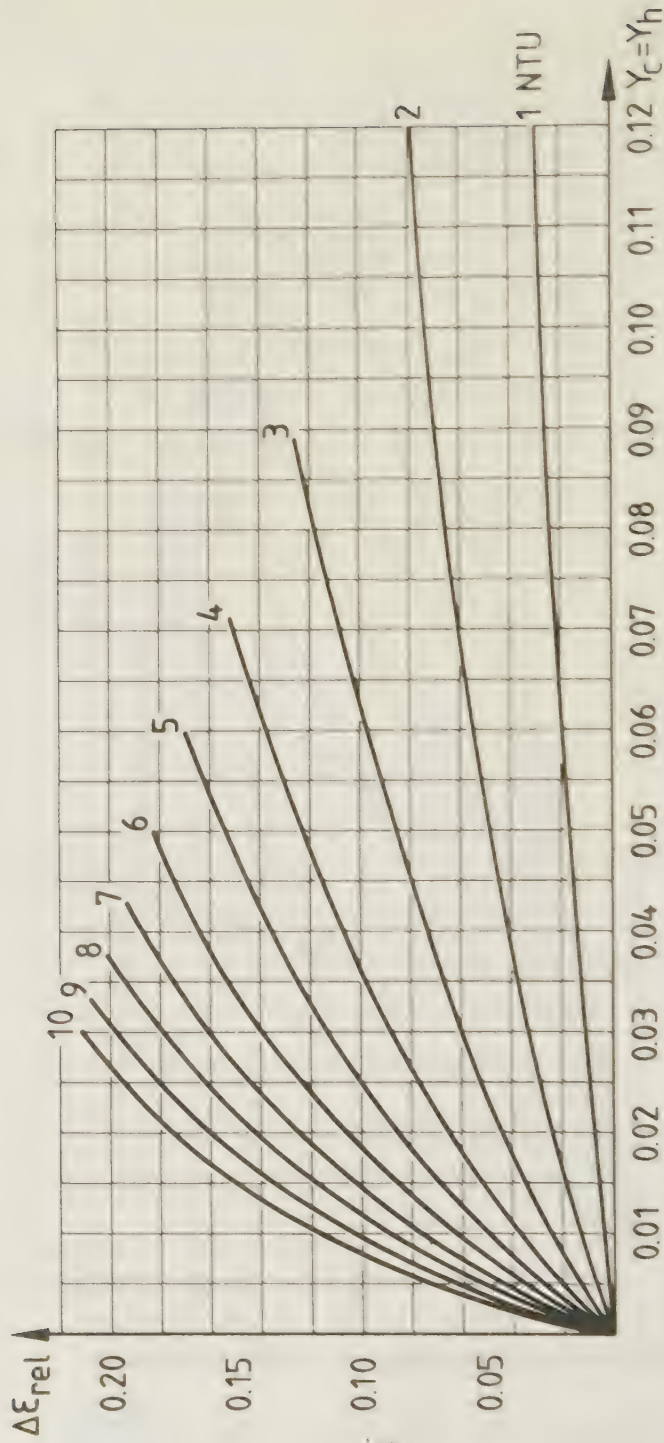


Fig 37. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

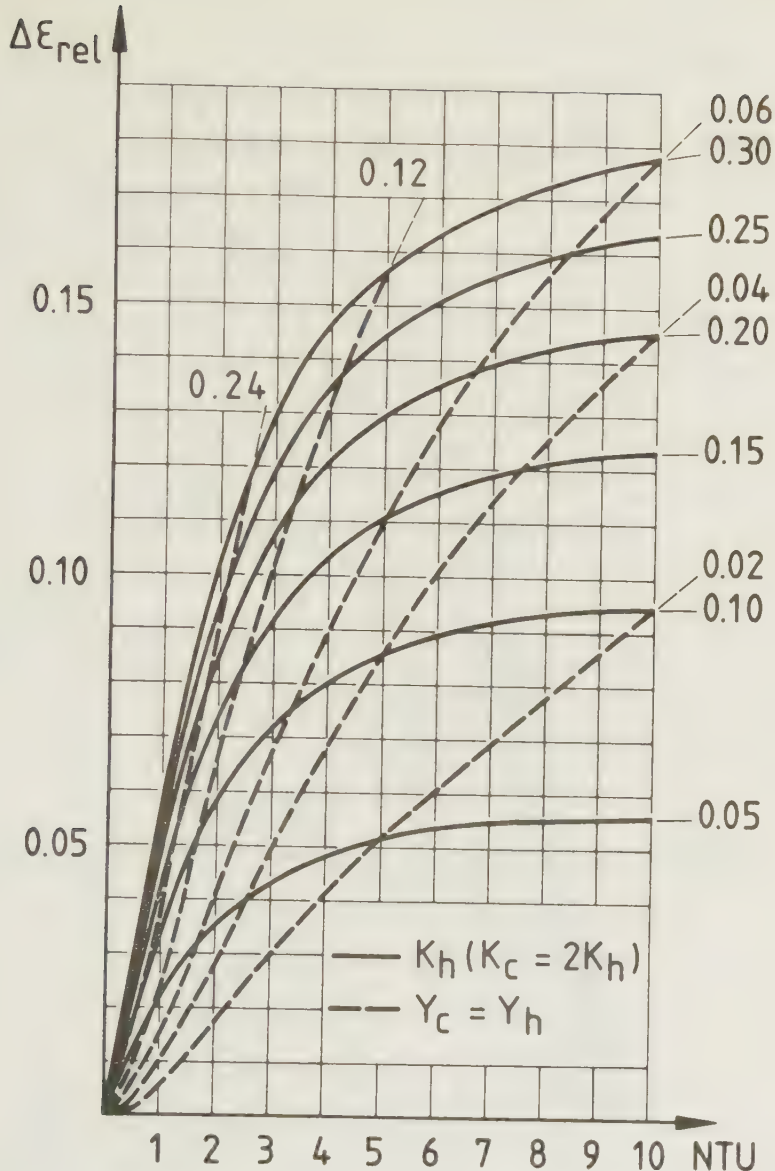


Fig 38. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 0.5$, $\frac{C_c}{C_h} = 0.5$.

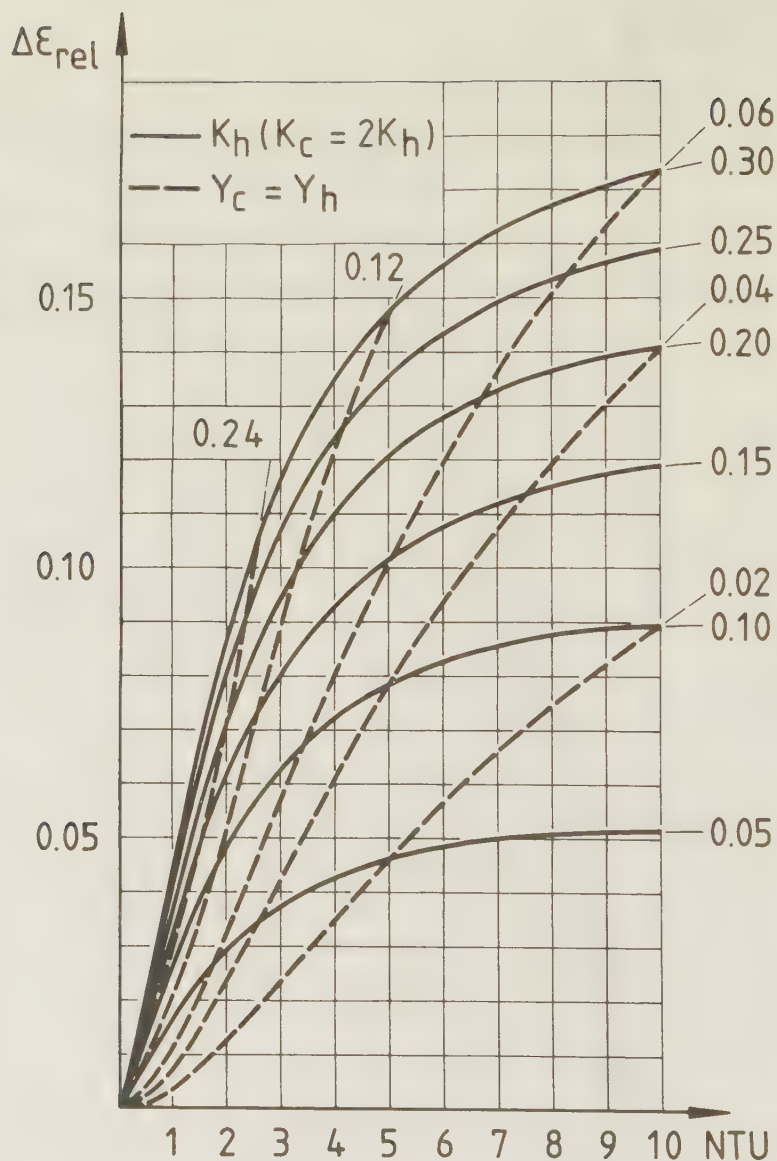


Fig 39. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 1.0$, $\frac{C_c}{C_h} = 0.5$.

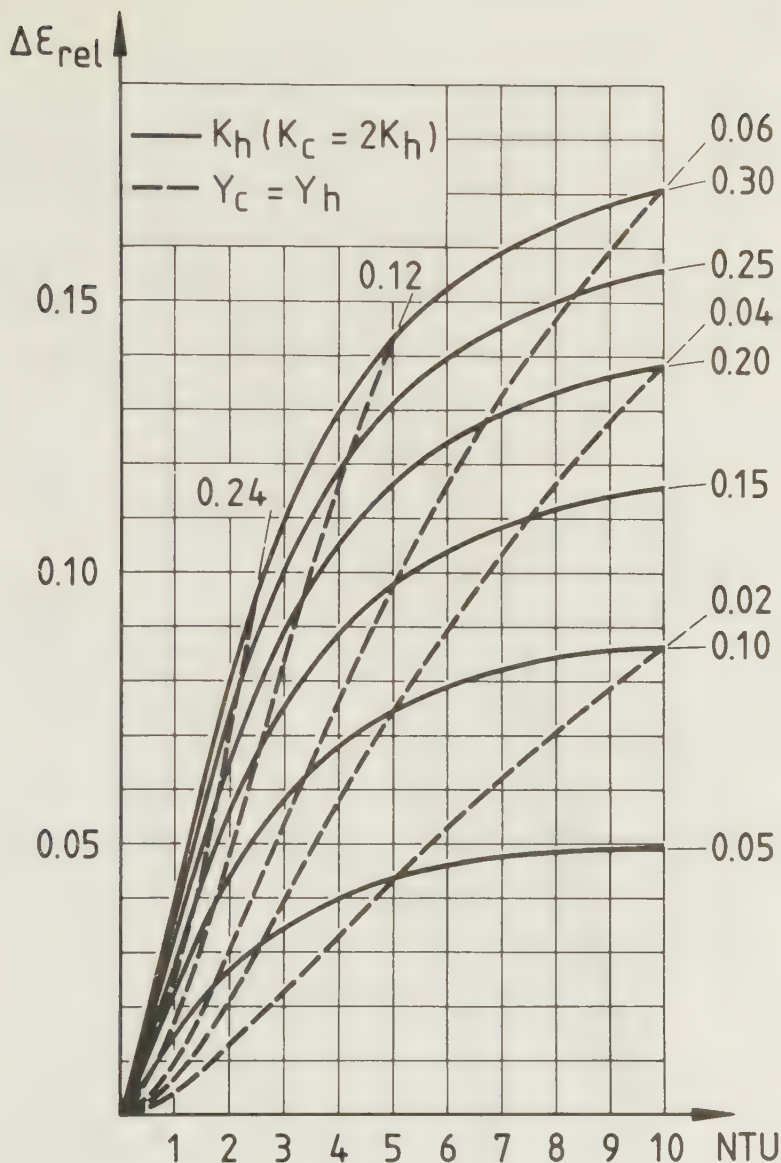


Fig 40. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 1.5$, $\frac{C_c}{C_h} = 0.5$.

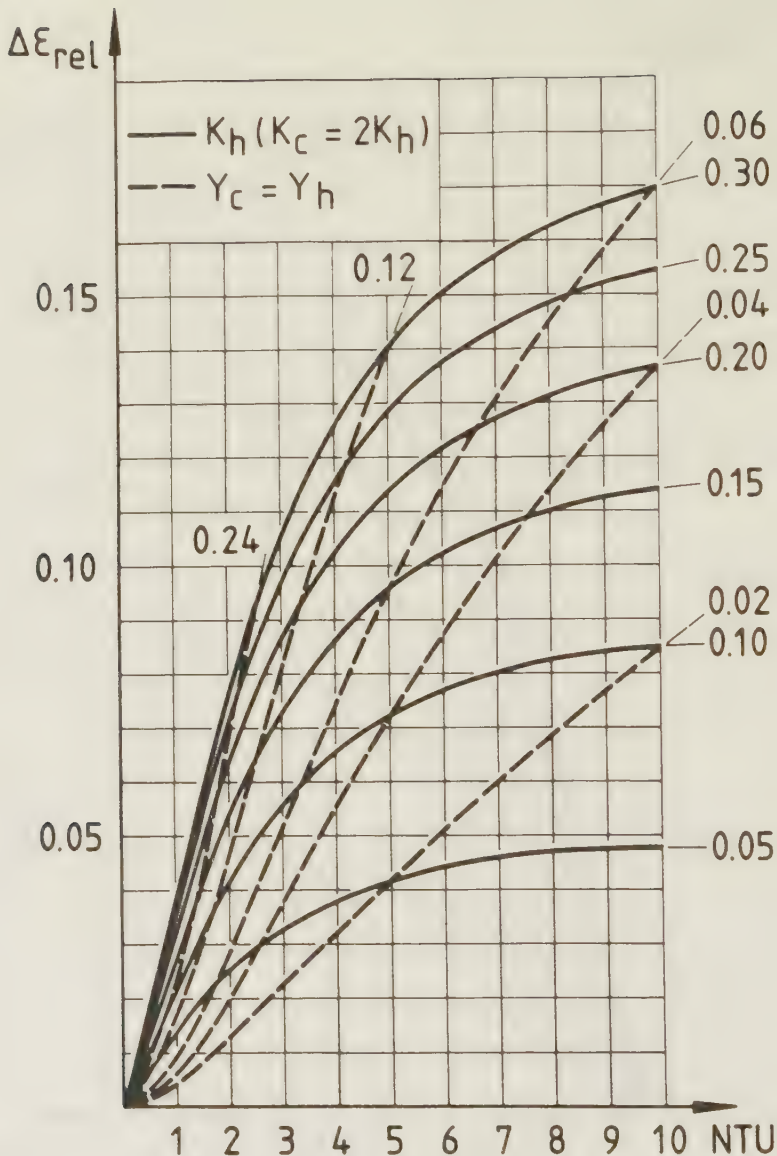


Fig 41. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 2.0$, $\frac{C_c}{C_h} = 0.5$.

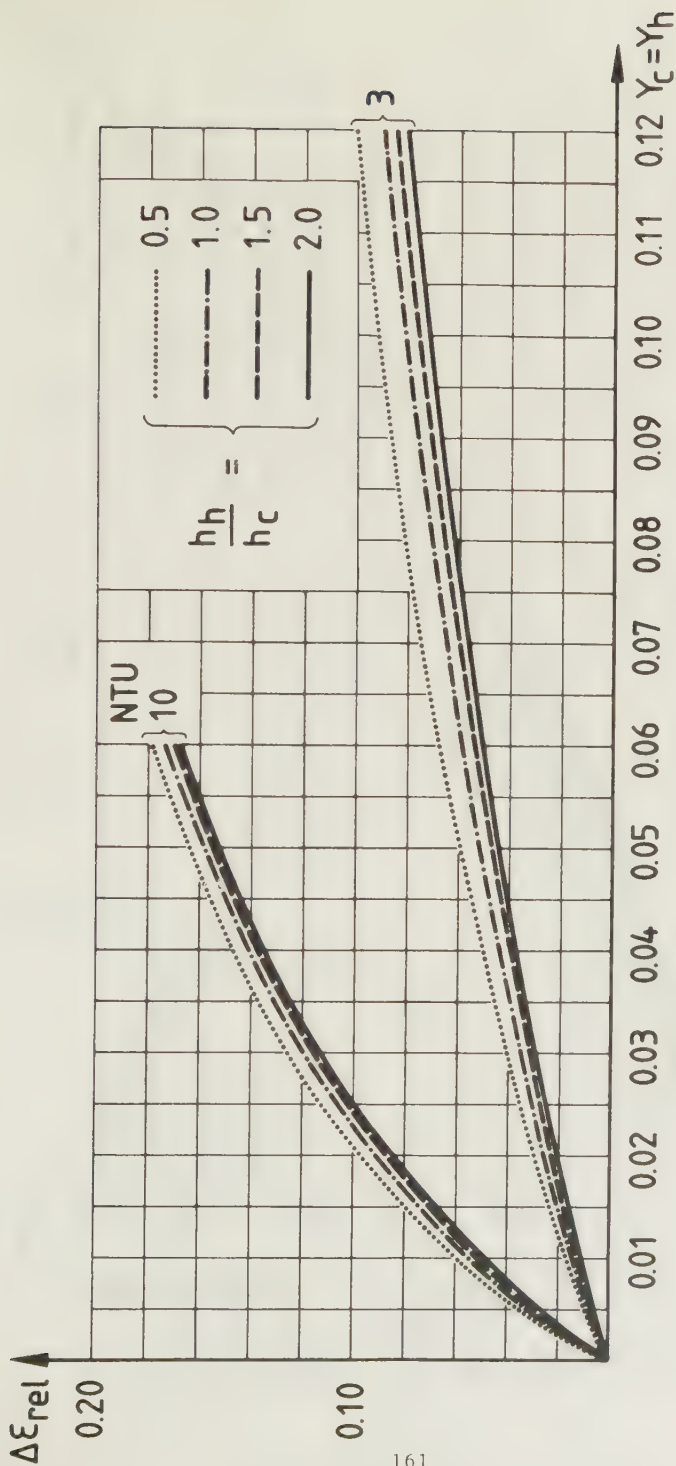


Fig 42. Relative decrease in effectiveness.

Single-pass, $\frac{C_c}{C_h} = 0.5$.

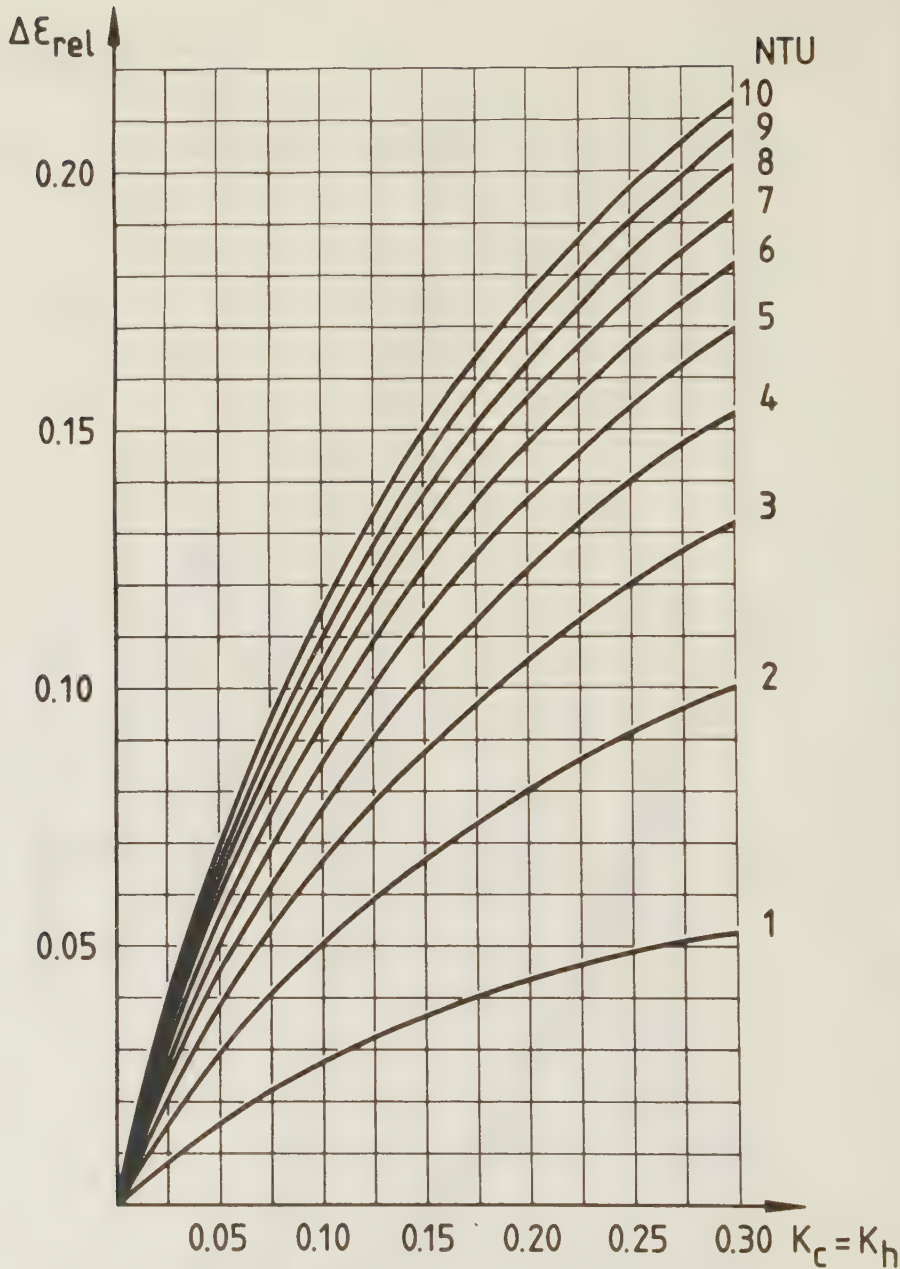


Fig 43. Relative decrease in effectiveness.

Single-pass, $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

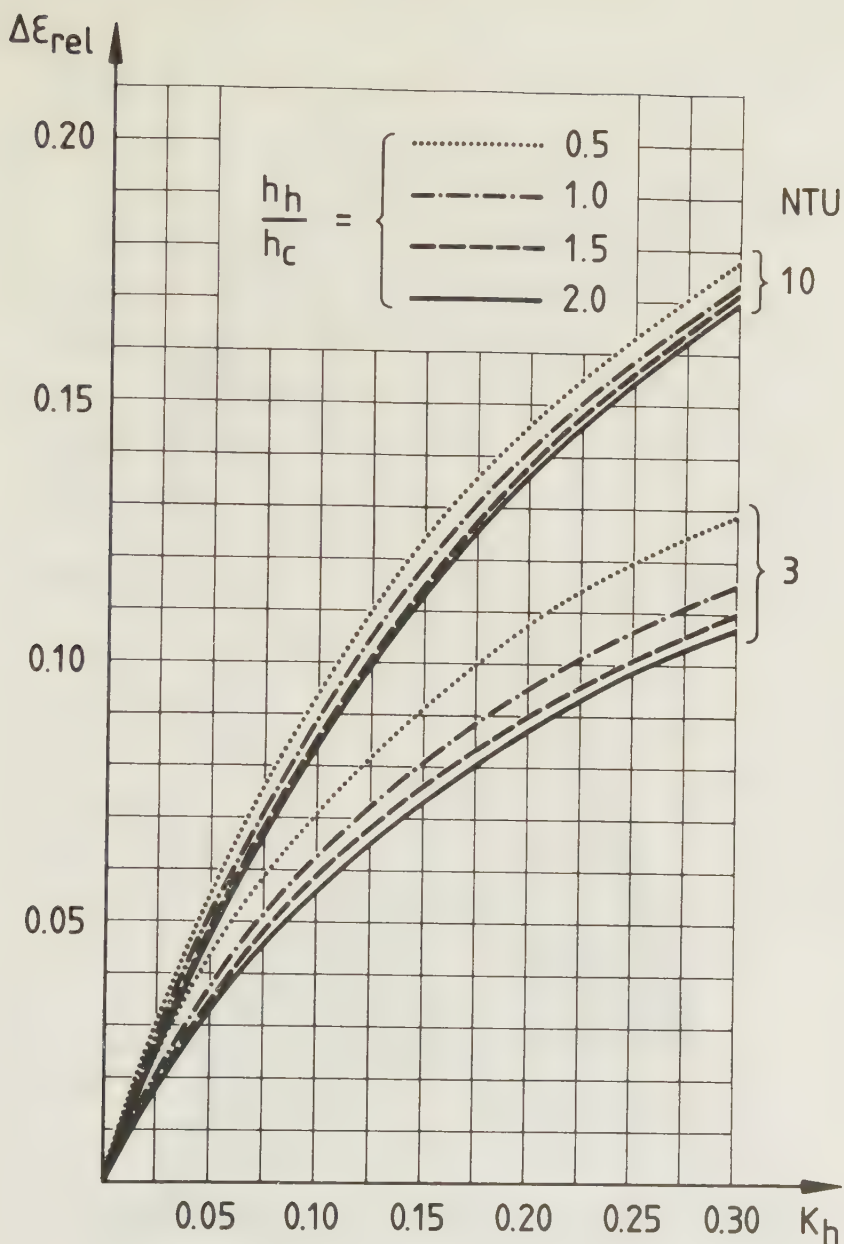


Fig 44. Relative decrease in effectiveness.

Single-pass, $\frac{C_c}{C_h} = 0.5$, $K_c = 2K_h$

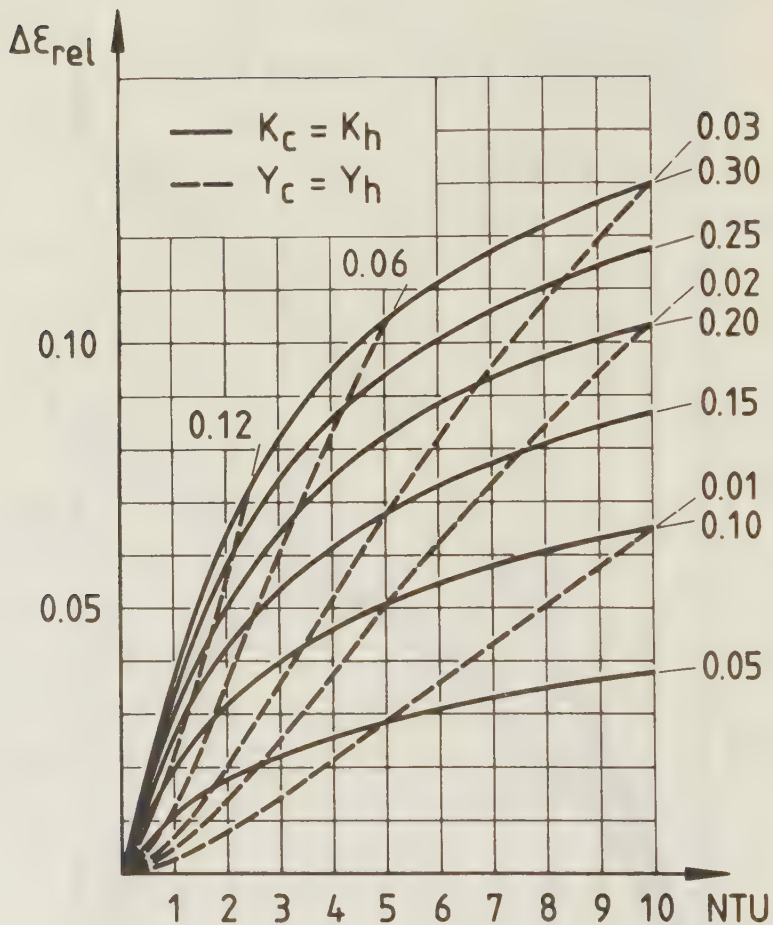


Fig 45. Relative decrease in effectiveness.

Two-pass, $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

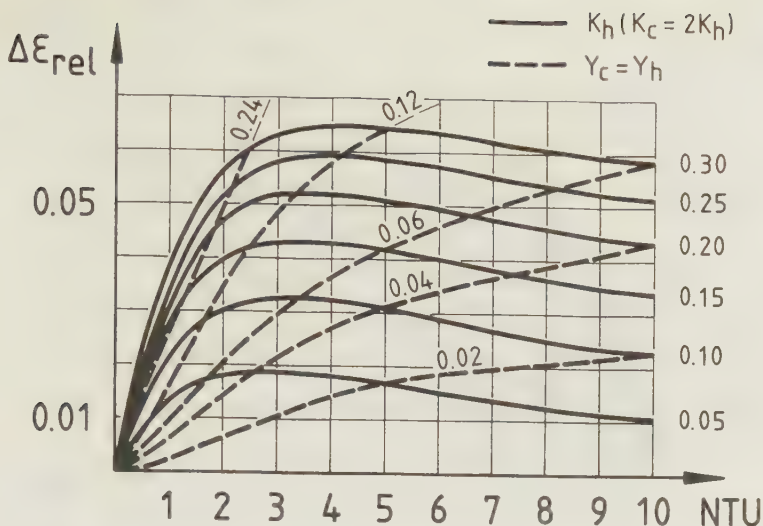


Fig 46. Relative decrease in effectiveness.

Two-pass, $\frac{h_h}{h_c} = 0.5$, $\frac{C_c}{C_h} = 0.5$.

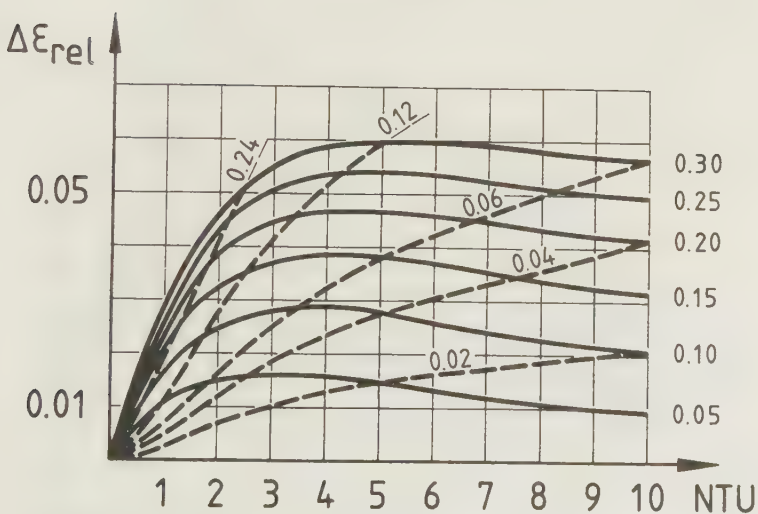


Fig 47. Relative decrease in effectiveness.

Two-pass, $\frac{h_h}{h_c} = 1.0$, $\frac{C_c}{C_h} = 0.5$.

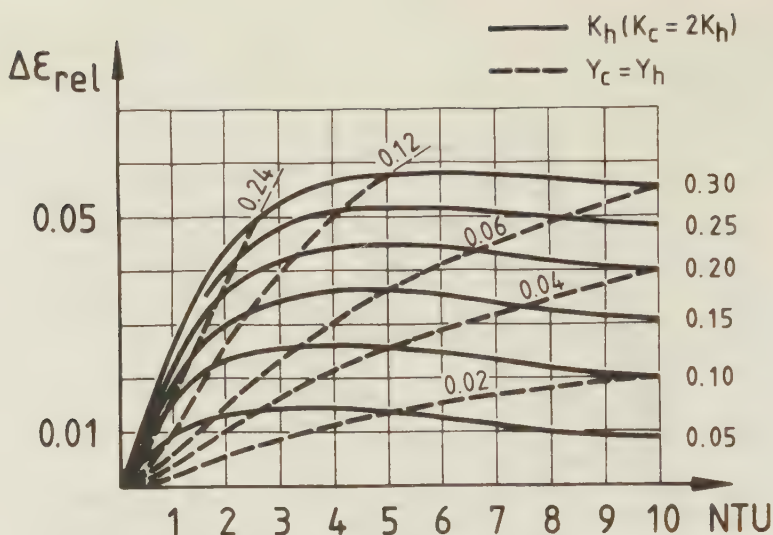


Fig 48. Relative decrease in effectiveness.

Two-pass, $\frac{h_h}{h_c} = 1.5$, $\frac{C_c}{C_h} = 0.5$.

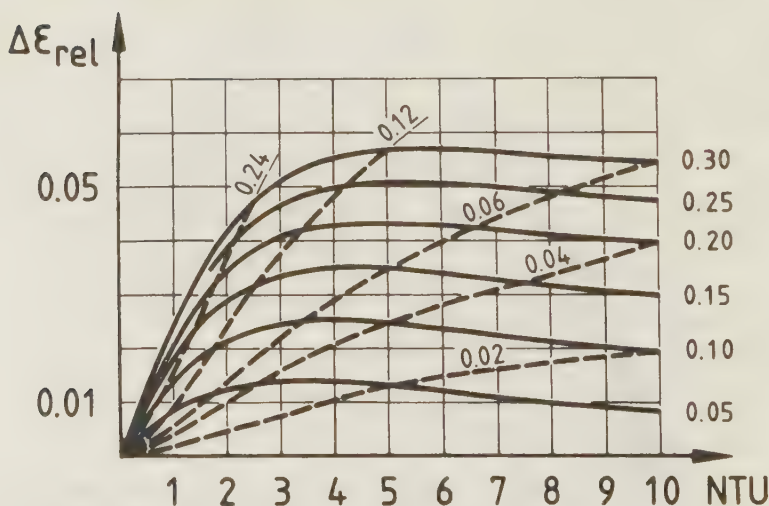


Fig 49. Relative decrease in effectiveness.

Two-pass, $\frac{h_h}{h_c} = 2.0$, $\frac{C_c}{C_h} = 0.5$.

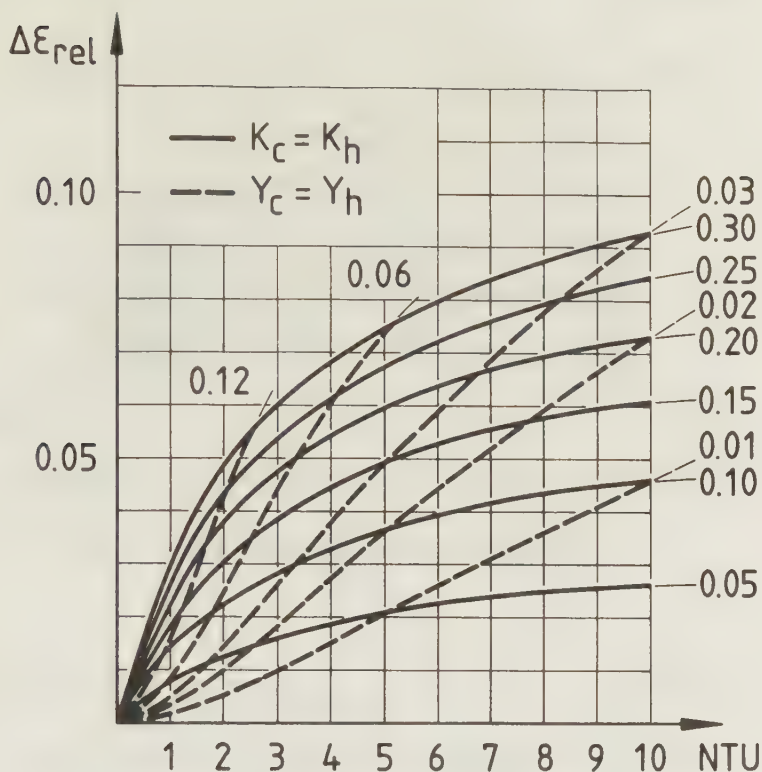


Fig 50. Relative decrease in effectiveness.

Three-pass, $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

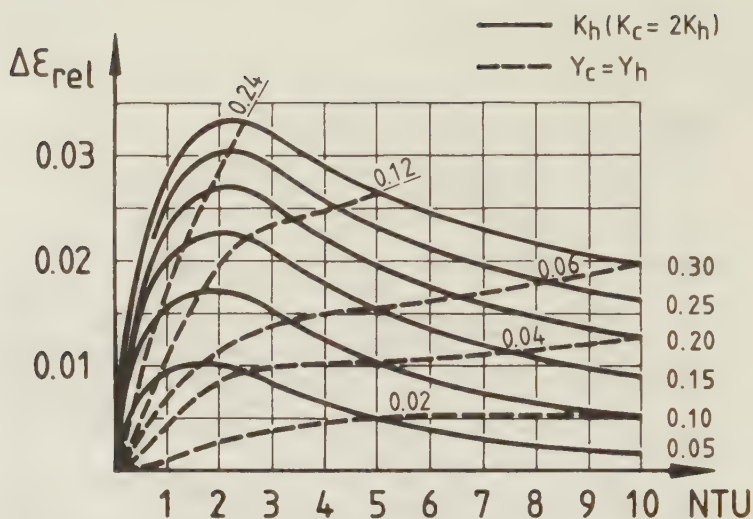


Fig 51. Relative decrease in effectiveness.

Three-pass, $\frac{h_h}{h_c} = 0.5$, $\frac{C_c}{C_h} = 0.5$.

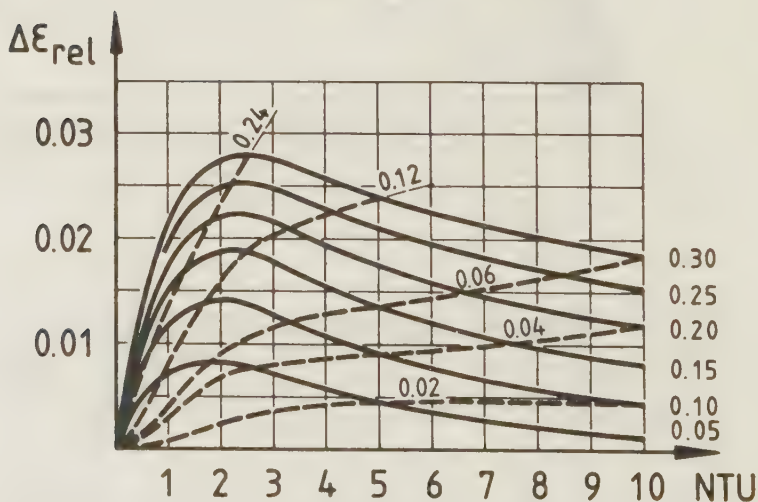


Fig 52. Relative decrease in effectiveness.

Three-pass, $\frac{h_h}{h_c} = 1.0$, $\frac{C_c}{C_h} = 0.5$.

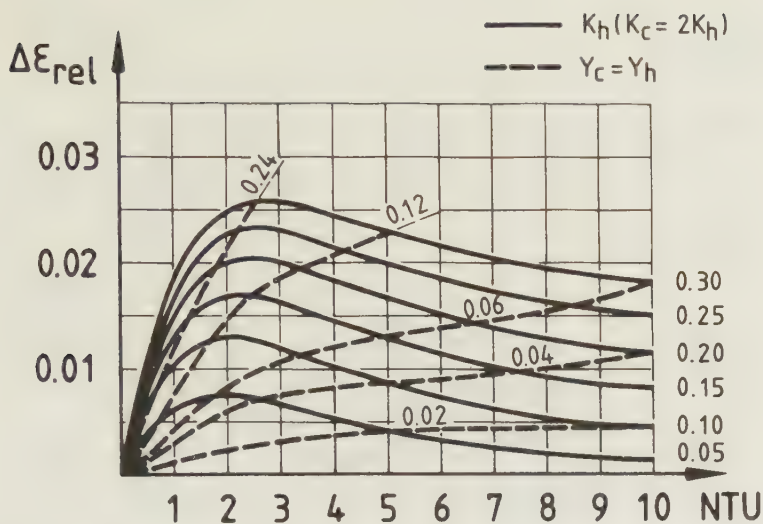


Fig 53. Relative decrease in effectiveness.

Three-pass, $\frac{h_h}{h_c} = 1.5$, $\frac{C_c}{C_h} = 0.5$.

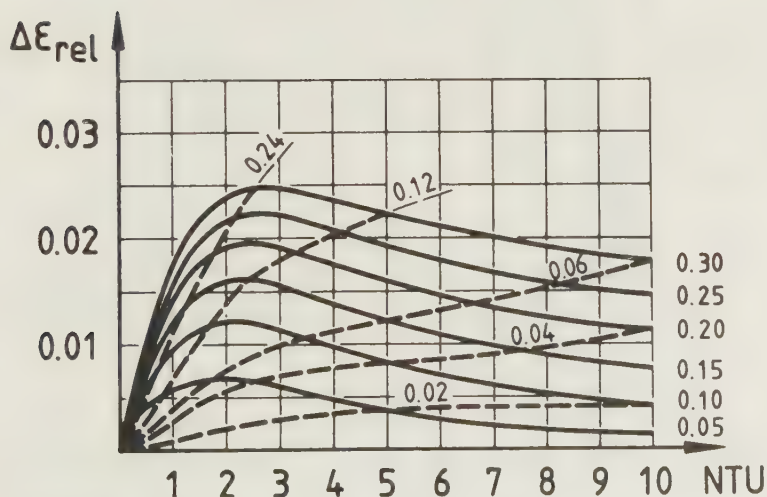


Fig 54. Relative decrease in effectiveness.

Three-pass, $\frac{h_h}{h_c} = 2.0$, $\frac{C_c}{C_h} = 0.5$.

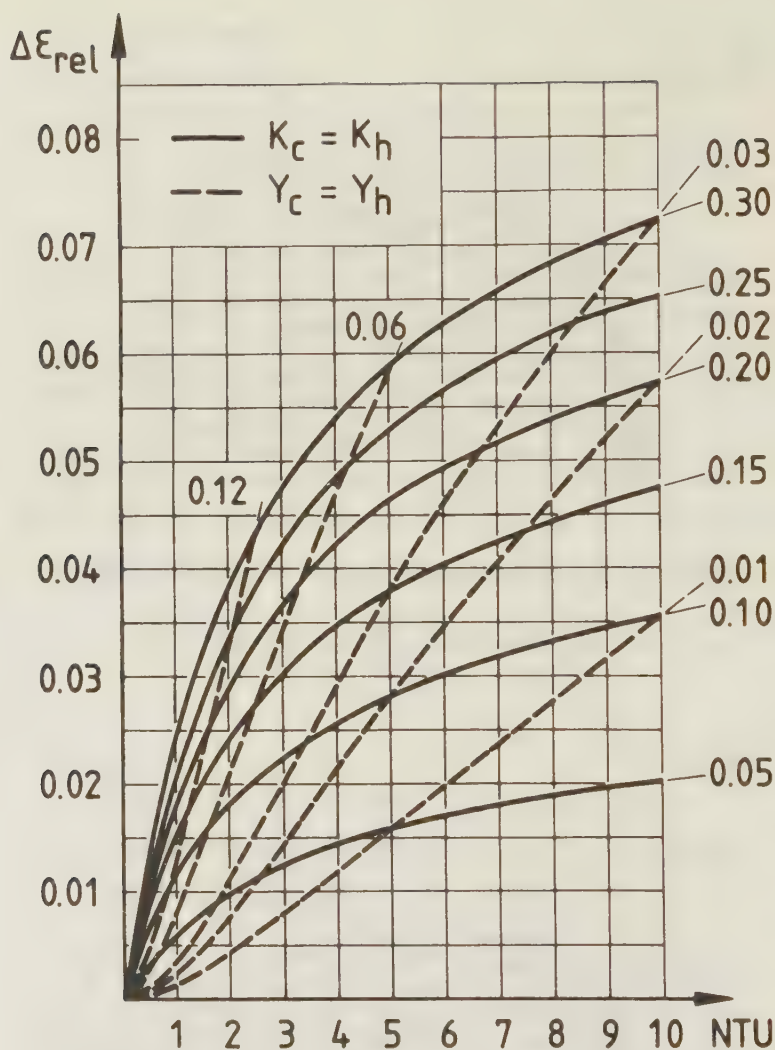


Fig 55. Relative decrease in effectiveness.

Four-pass, $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

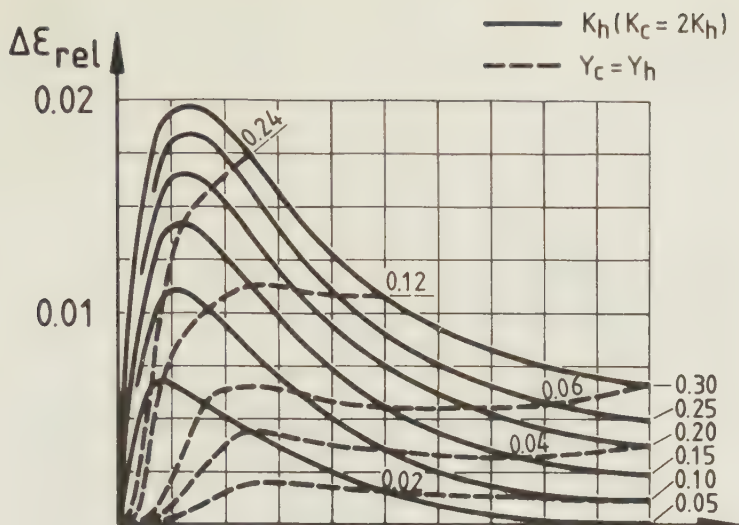


Fig 56. Relative decrease in effectiveness.

Four-pass, $\frac{h_h}{h_c} = 0.5$, $\frac{C_c}{C_h} = 0.5$.

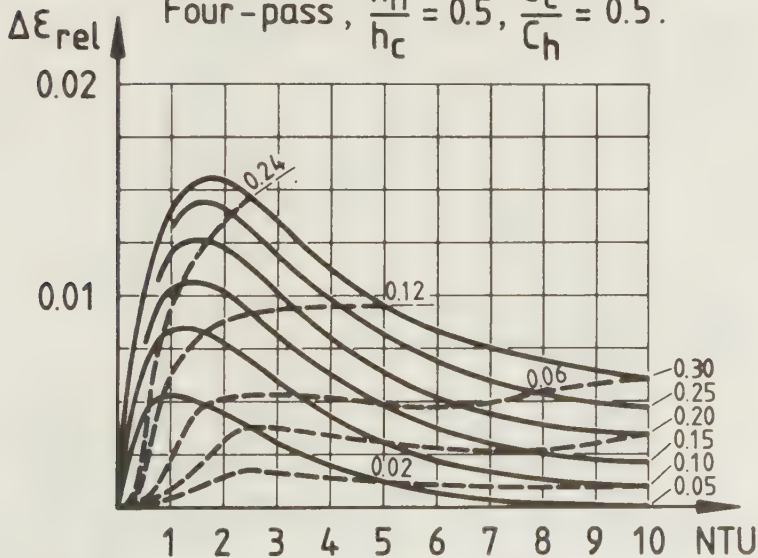


Fig 57. Relative decrease in effectiveness.

Four-pass, $\frac{h_h}{h_c} = 1.0$, $\frac{C_c}{C_h} = 0.5$.

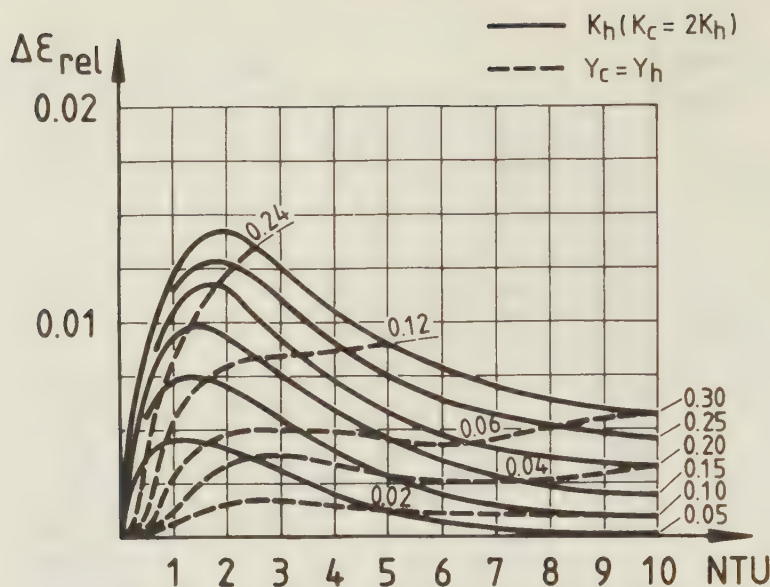


Fig 58. Relative decrease in effectiveness.

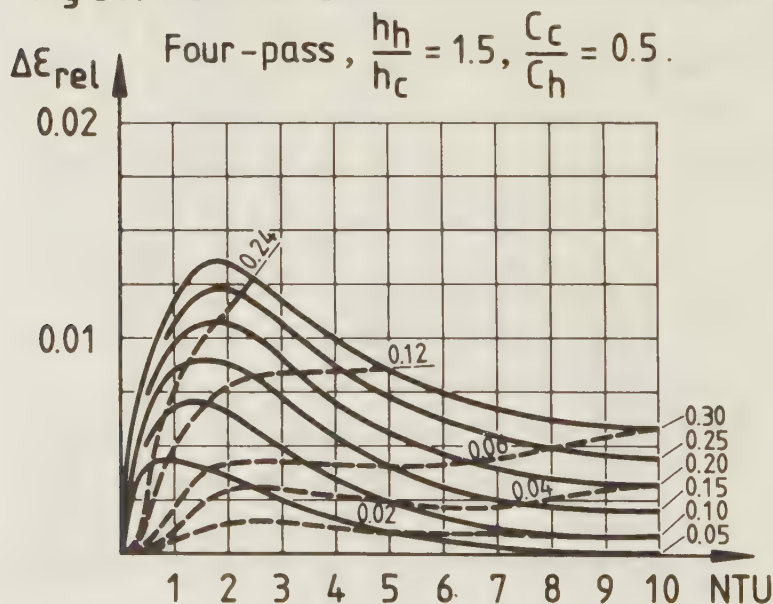


Fig 59. Relative decrease in effectiveness.

Four-pass, $\frac{h_h}{h_c} = 2.0, \frac{C_c}{C_h} = 0.5.$

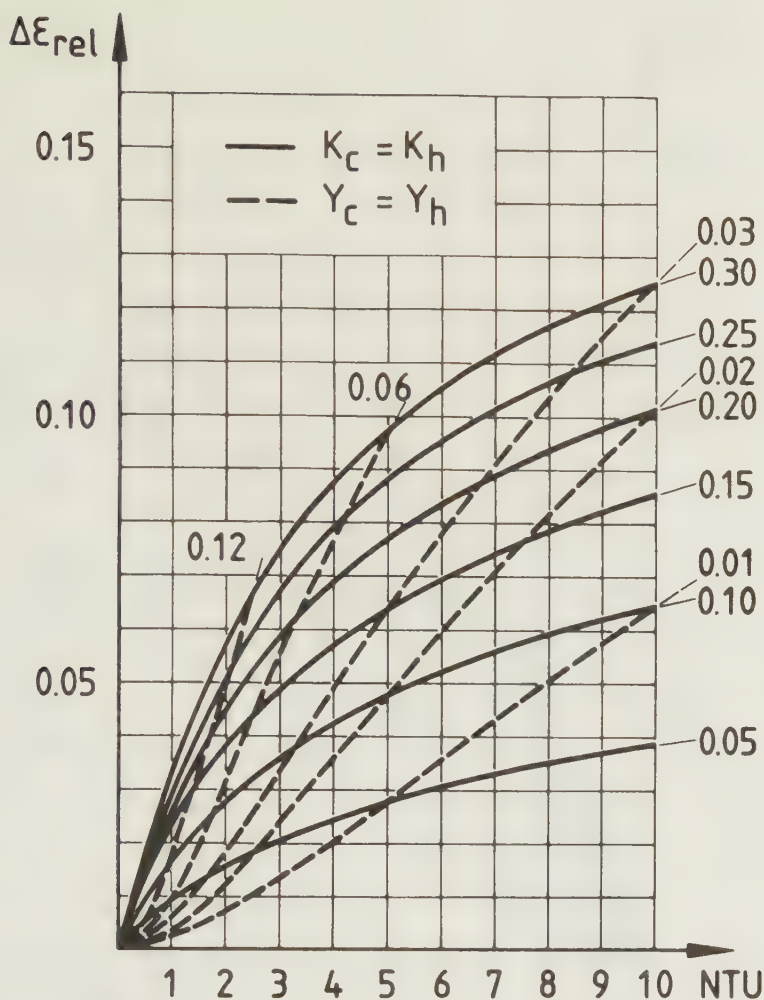


Fig 60. Relative decrease in effectiveness.

Two-pass, hot side mixed between passes,
 $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

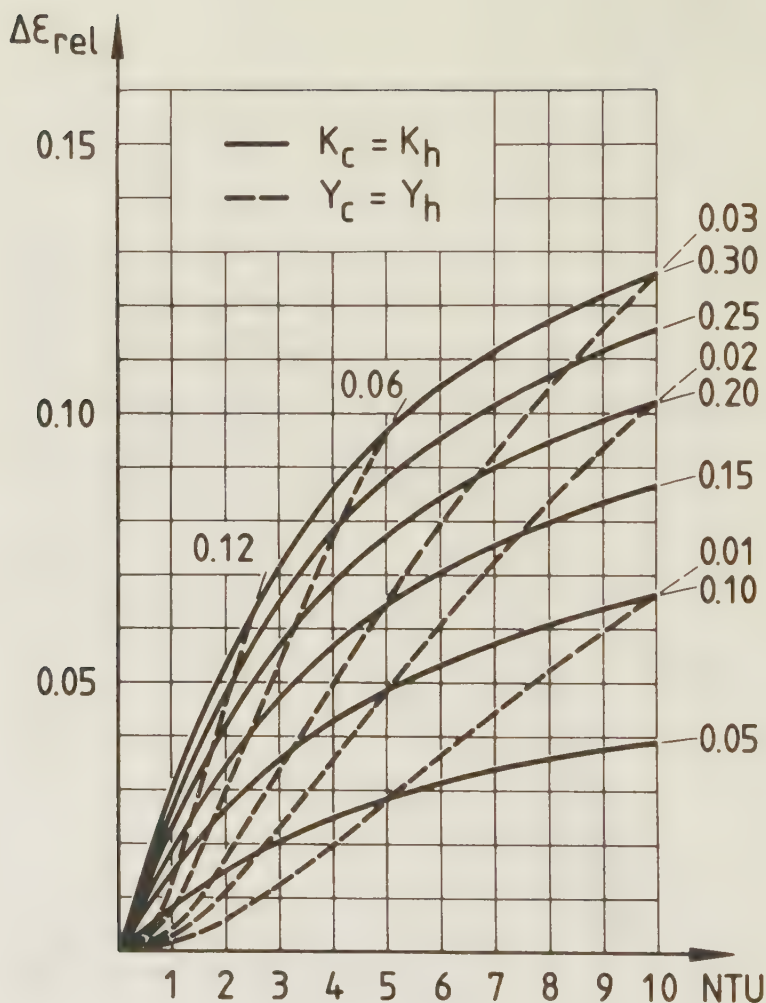


Fig 61. Relative decrease in effectiveness.

Two-pass, no mixing between passes,
 $\frac{h_h}{h_c} = 0.5 - 2.0$, $\frac{C_c}{C_h} = 1.0$.

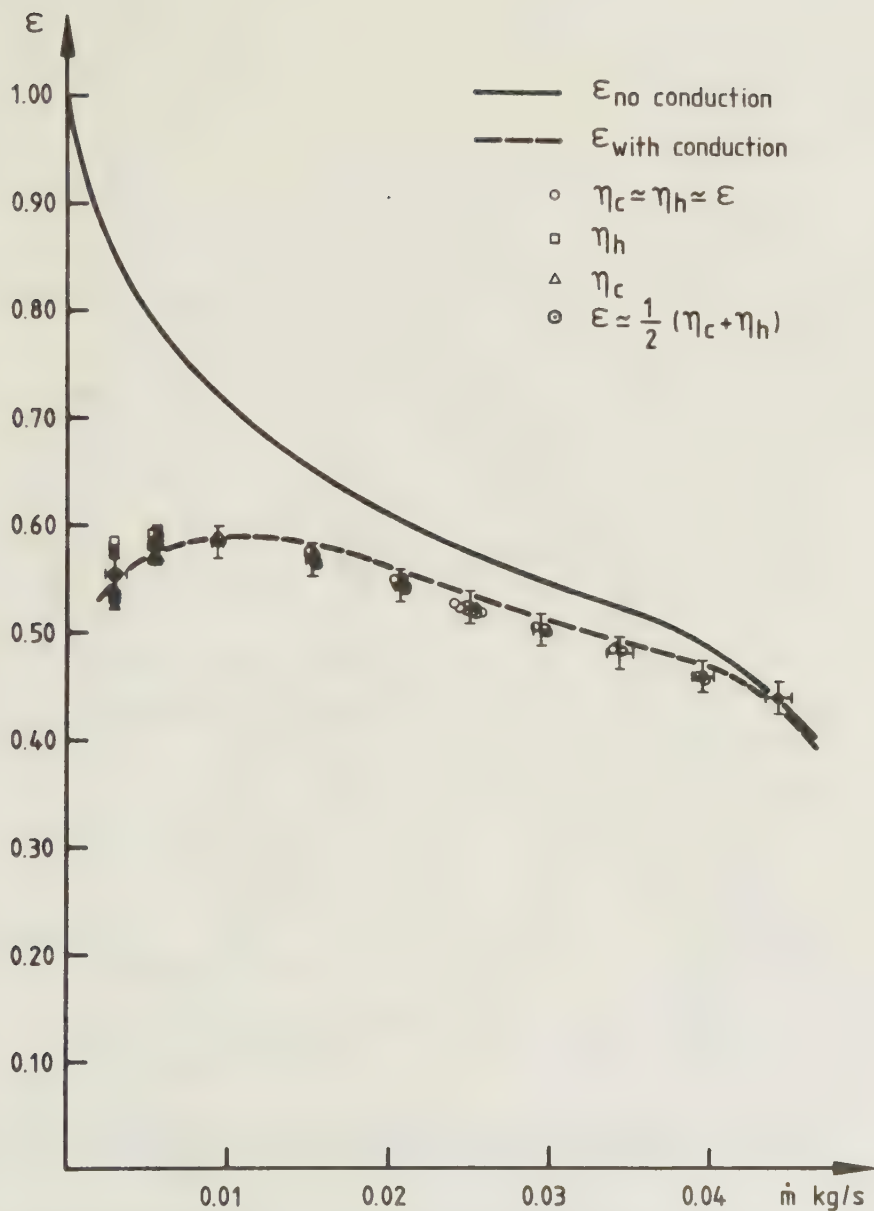


Fig 62a. Results of tests and calculations from tested crossflow heat exchanger.

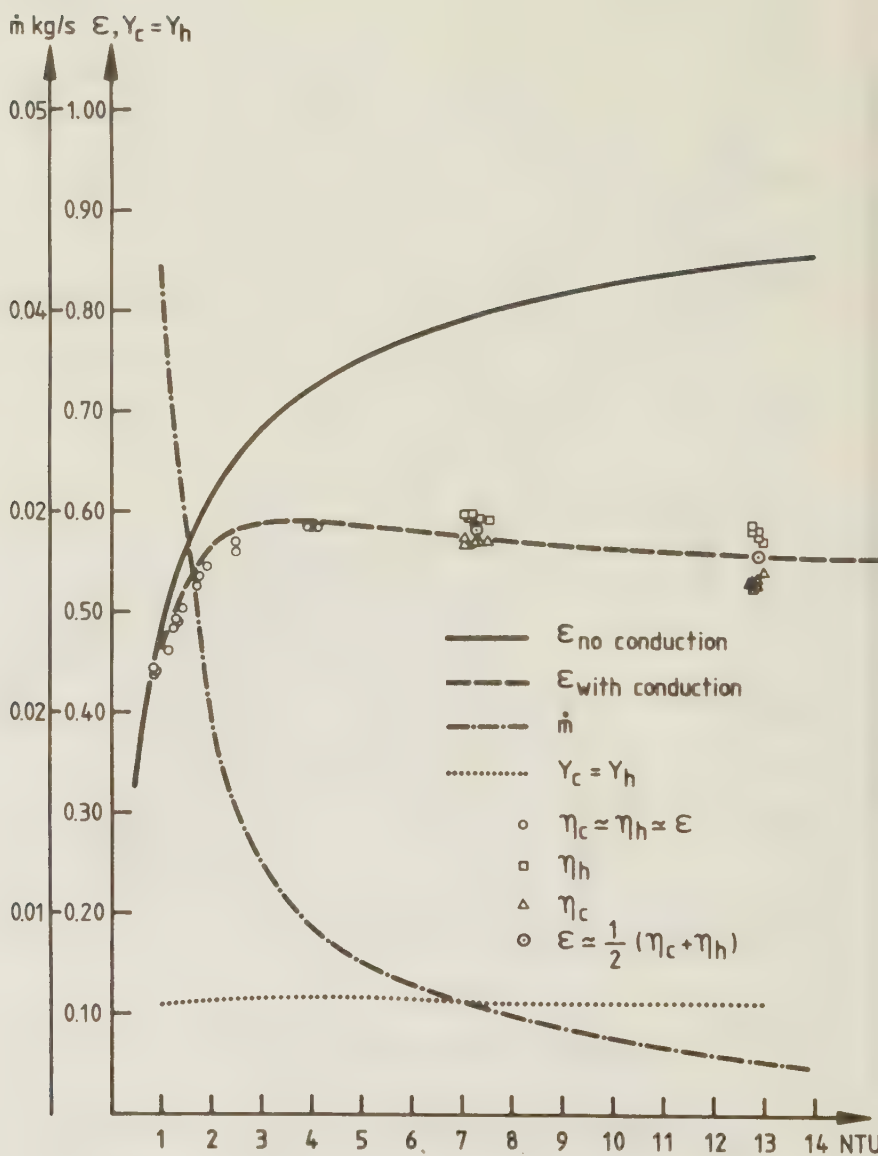


Fig 62b. Results of tests and calculations from tested crossflow heat exchanger.

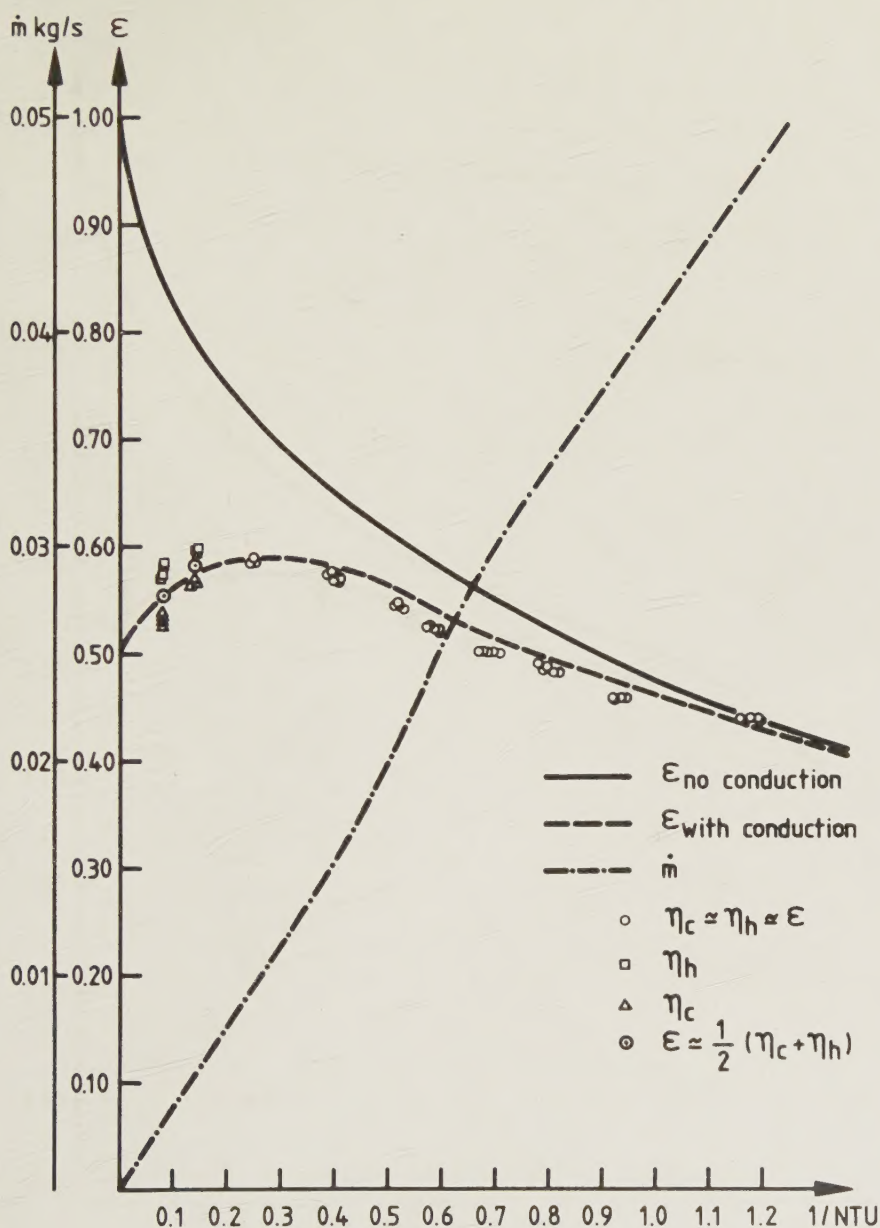


Fig 62 c. Results of tests and calculations from tested crossflow heat exchanger.



Fig. 8. Results of tests and correlations for heat transfer in a cross-section of a tube.

